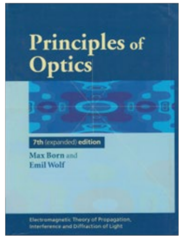
Seamless transition to geometrical optics concepts in a fully physical optics framework

Abstract

VirtualLab Fusion is a software product developed for multiscale optical simulations and design. It offers a platform that interconnects different simulation models, such as precise Maxwell solvers, like the Fourier Modal Method (FMM), and more approximate methods, including geometrical optics. The ability for various simulation models to work together requires that all methods utilize electromagnetic fields as both input and output. Therefore, integrating geometrical optics into a multiscale simulation framework necessitates developing a formulation of geometrical optics for electromagnetic fields.This document examines the theory, which is also applied within VirtualLab Fusion.

The document originates from a presentation delivered by Frank Wyrowski at SPIE Europe in April 2024.

We need to identify that part of physical optics, which deals with the "***geometrical laws relating to the propagation of the 'amplitude vectors' E and H.***"

Citation from 

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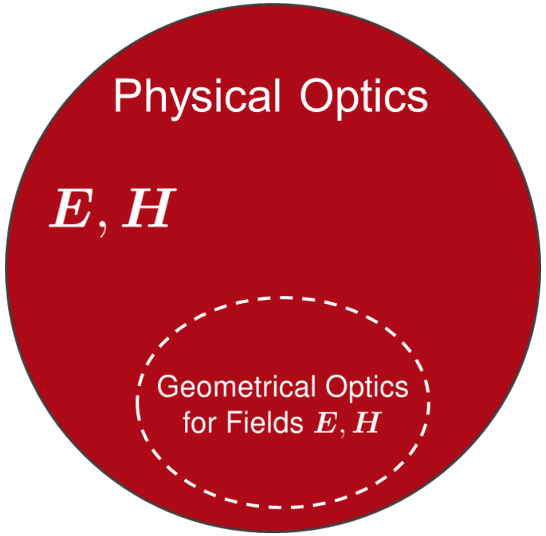
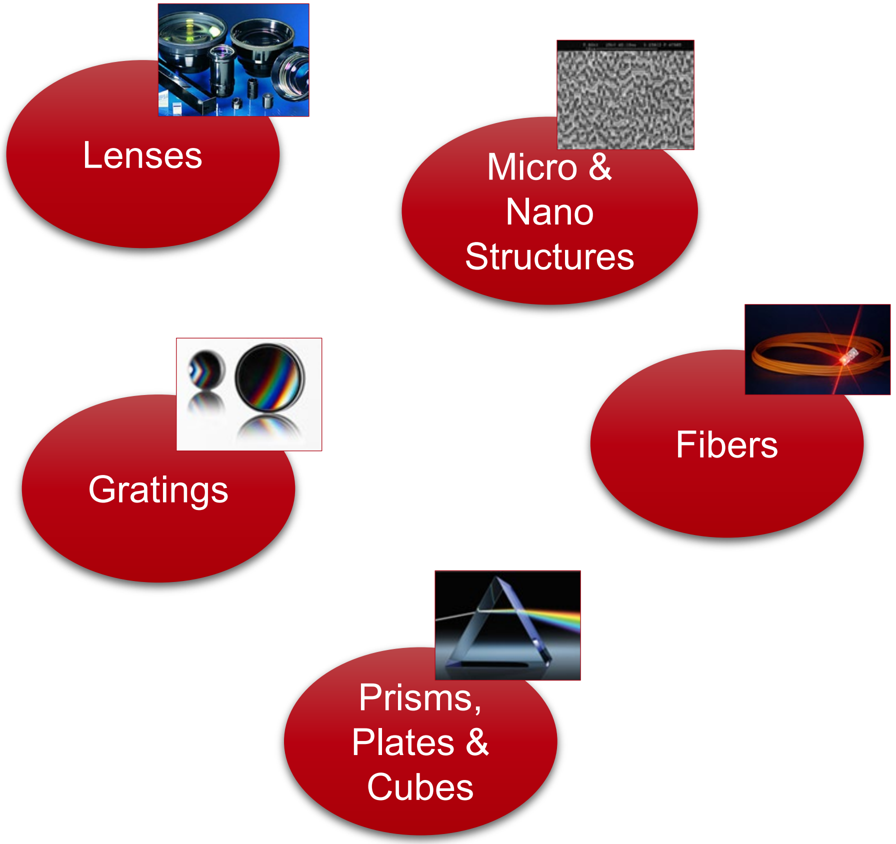


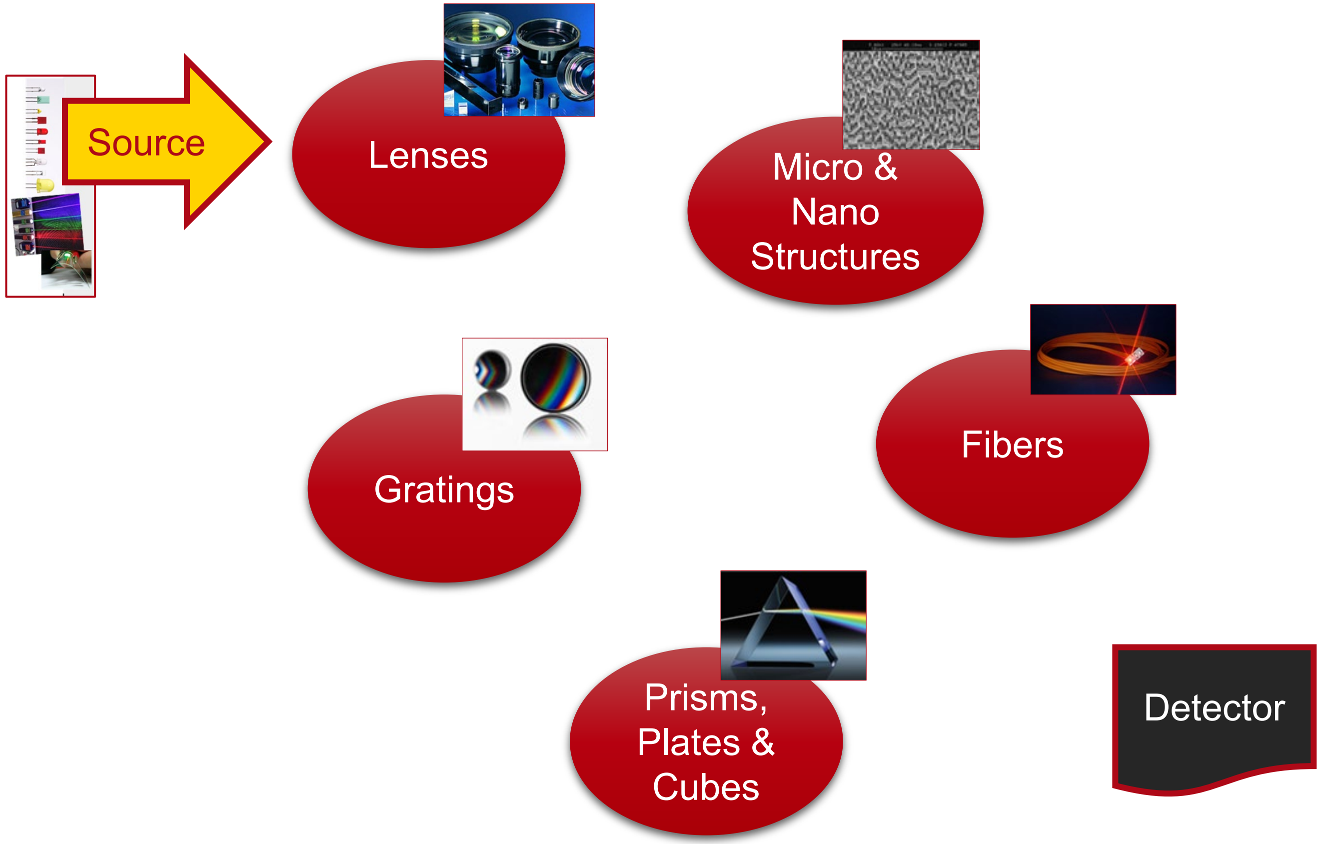
Simulating Optical Systems



Systems are composed of a continuously expanding variety of components, such as lenses, freeform surfaces, Fresnel lenses, pancake lenses, GRIN lenses, metalenses, mirrors, gratings, diffractive optical elements (DOEs), crystals, apertures, prisms, cubes, fibers, scatterers, diffusers, micro lens arrays, and spatial light modulators (SLMs).

The simulation must be appropriate for a multiscale system configuration.

Simulating Optical Systems



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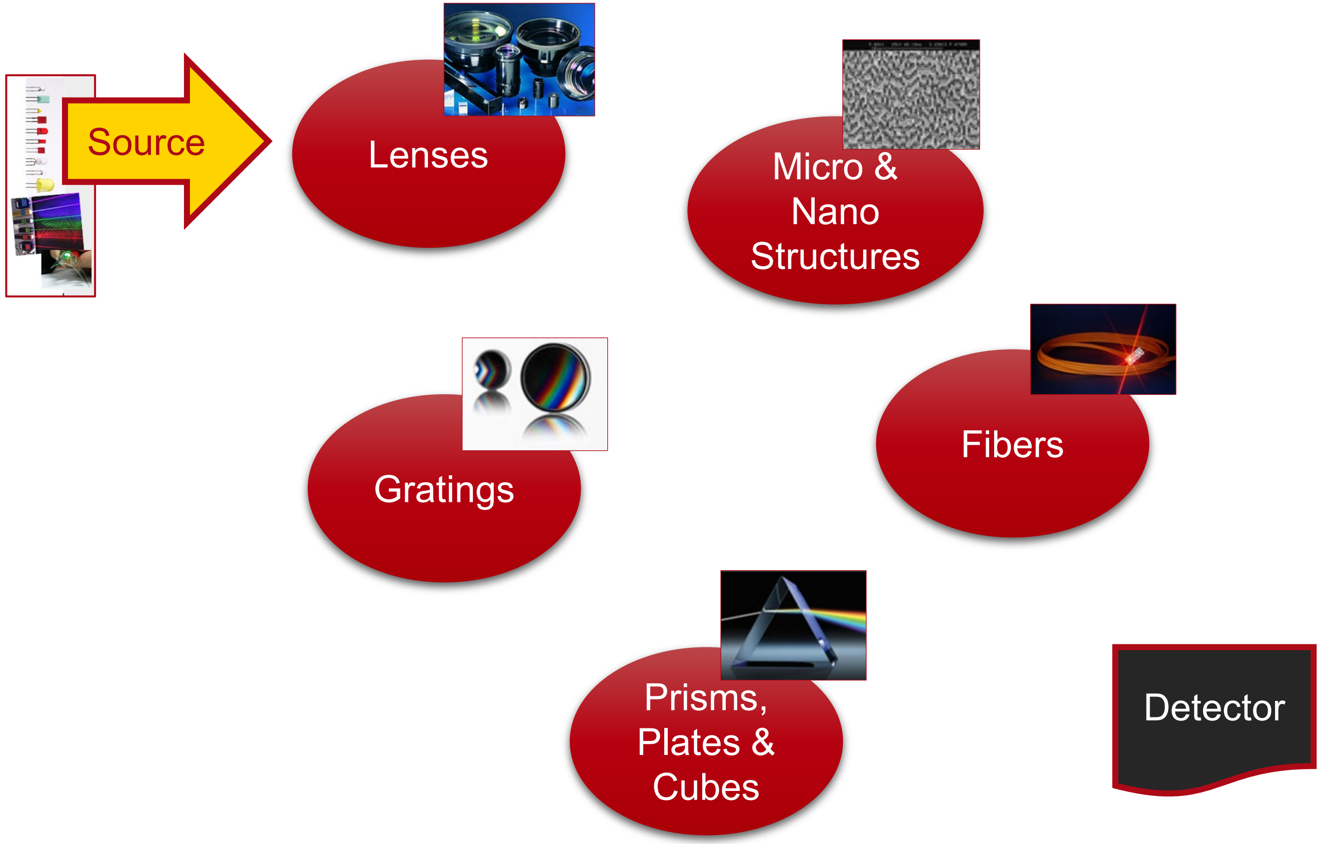
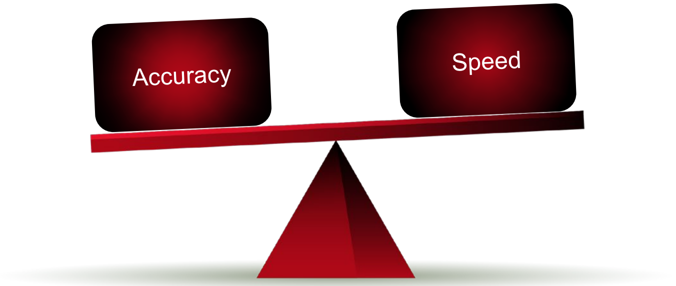
The simulation should generate all required detector outputs, such as aberrations, point spread function (PSF), modulation transfer function (MTF), beam characteristics, radiometry, photometry, colorimetry, and diagnostics for ultrashort pulses.

The simulation must be appropriate for a multiscale system configuration.

Finding the Right Balance Between Accuracy and Speed

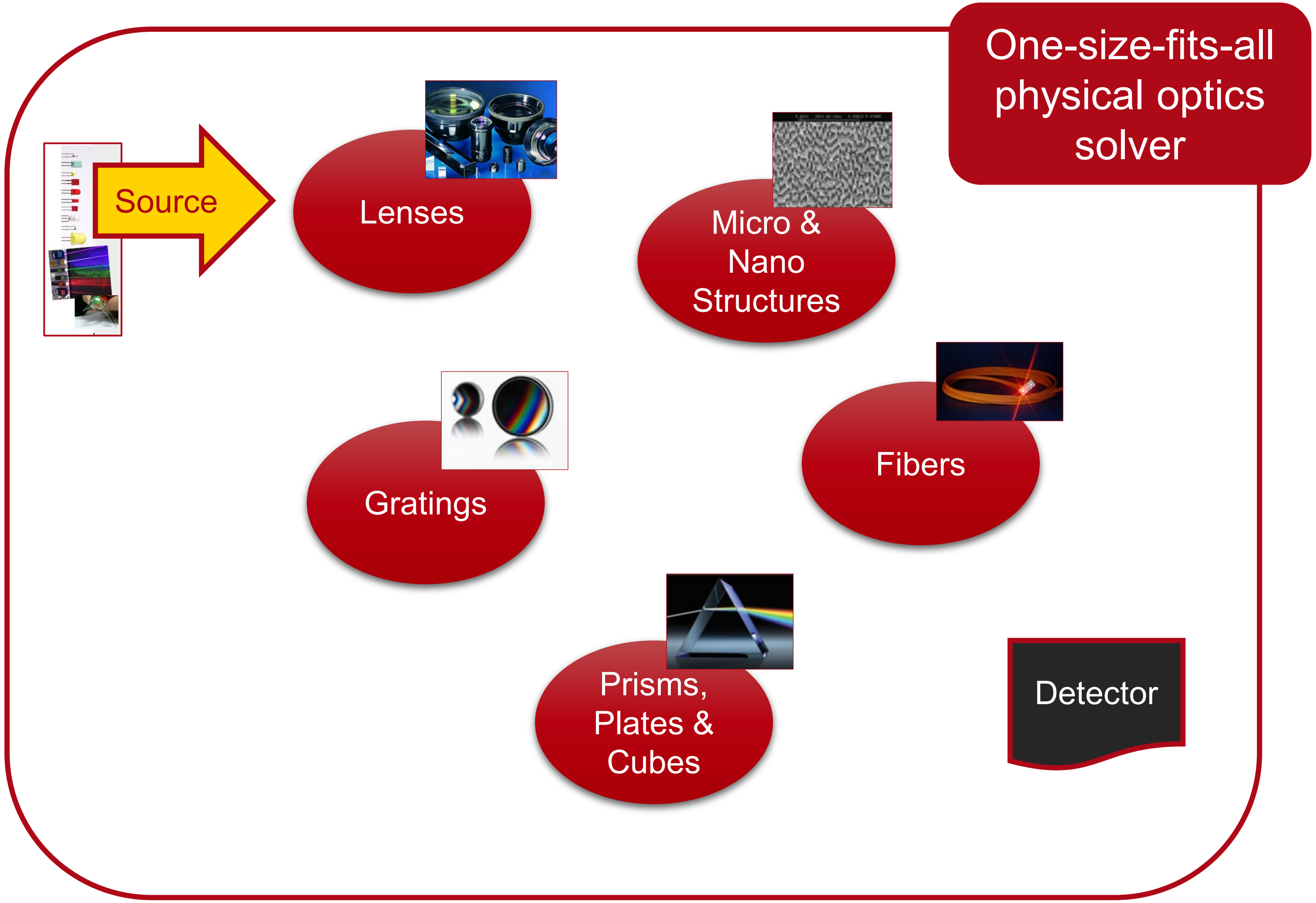


Simulations should aim to achieve the necessary level of accuracy while also being computationally efficient.



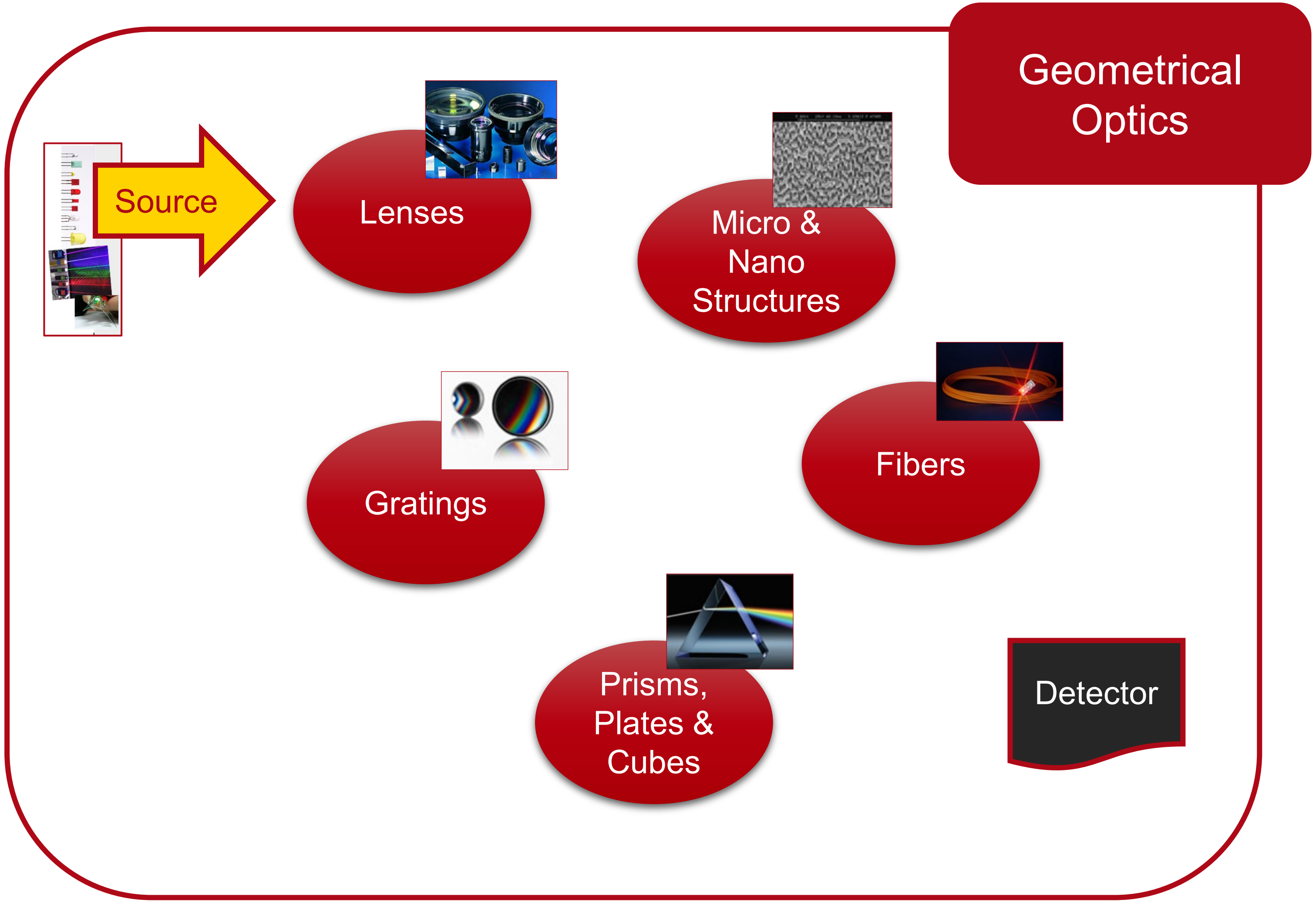
Control of accuracy-speed balance

Simulating Optical Systems: Universal Solver



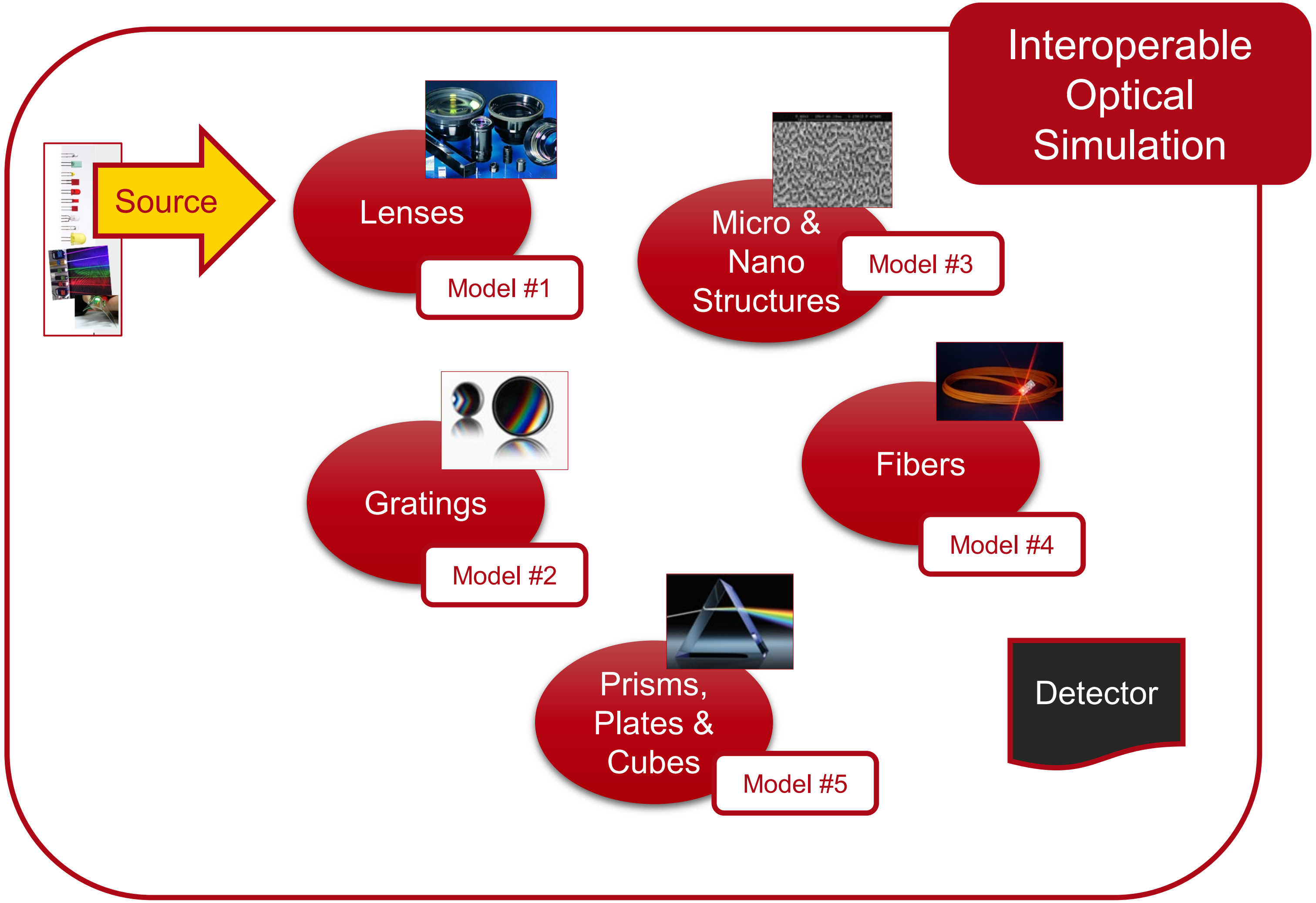
- Solvers that can be used for solving Maxwell's equations universally include methods such as:
  - Fourier Modal Method (FMM),
  - Finite Difference Time Domain Method (FDTD),
  - Finite Element Method (FEM).
- While these solvers are highly precise, they tend to be slow and demand significant computational resources.
- Their significance lies in their ability to simulate the impact of nanostructures.
- **Universal solvers are not appropriate for system simulation.**

Simulating Optical Systems: Geometrical Optics



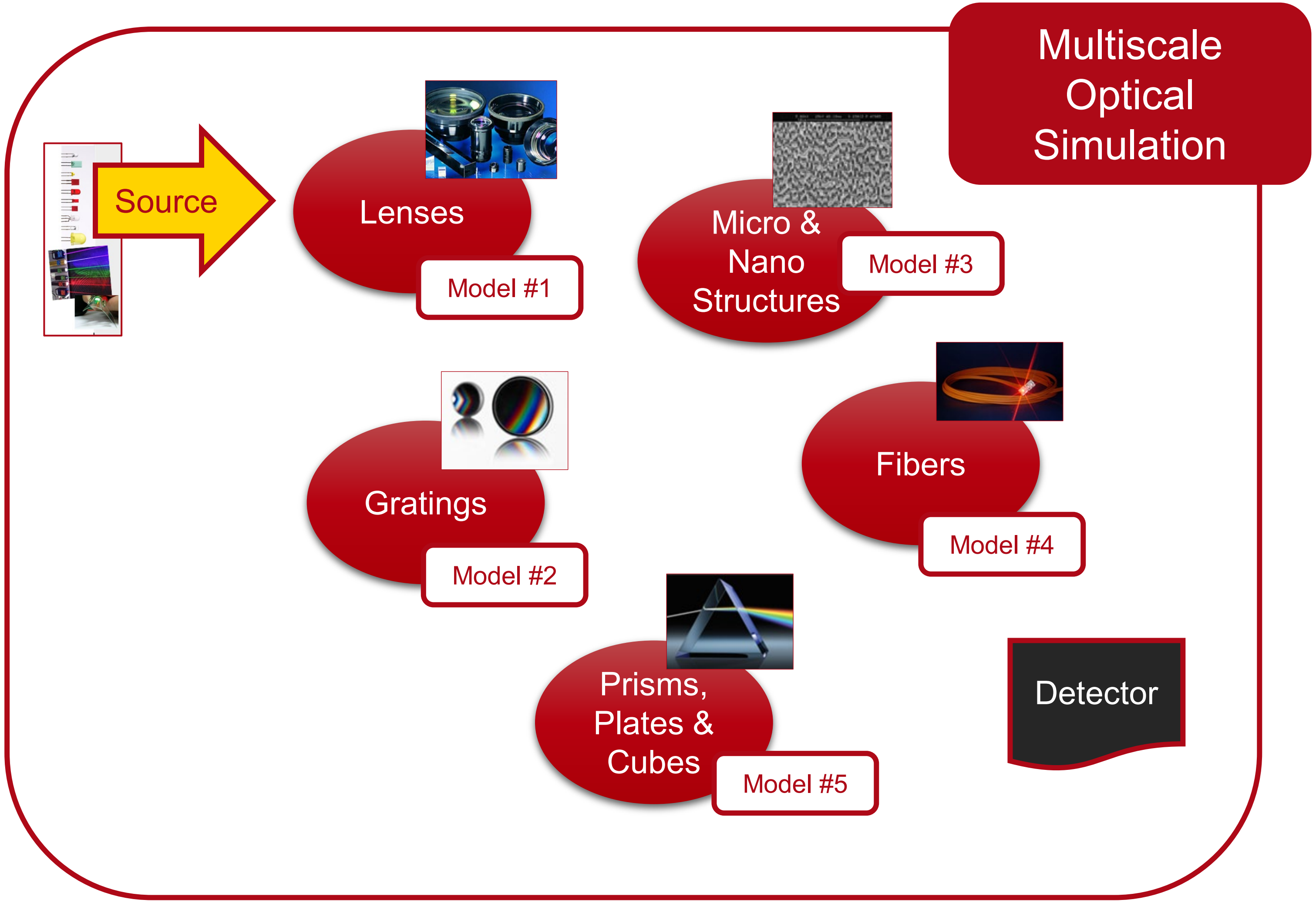
- Utilizing geometric optics in simulations often leads to quick results.
- It allows for the precise assessment of aberrations in lens systems.
- In general, the precision of geometrical optics is viewed as being moderate.
- **Geometrical optics is not sufficient to model the wide variety of optical systems.**

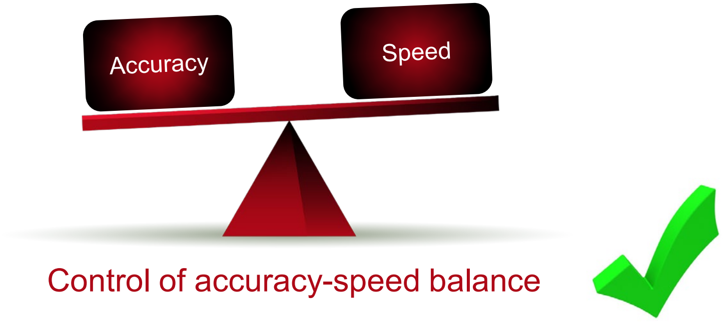
Simulating Optical Systems: Combine Many Techniques

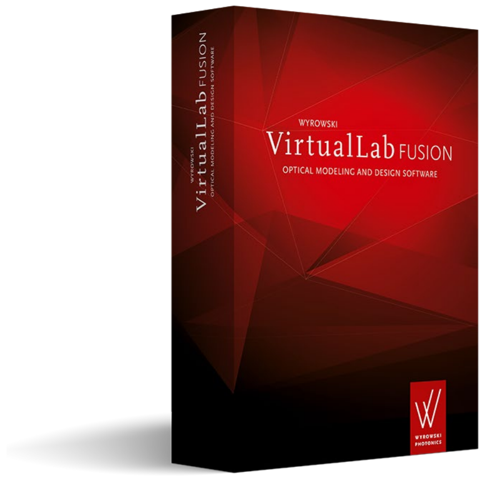


- Utilize customized simulation models for each component.
- Simulation models range from **geometrical optics to rigorous and simplified methods in physical optics**.
- For effective system modeling, it is crucial that all **methods are interoperable** with each other to enable seamless integration.

Multiscale Optical Simulation by VirtualLab Fusion Software







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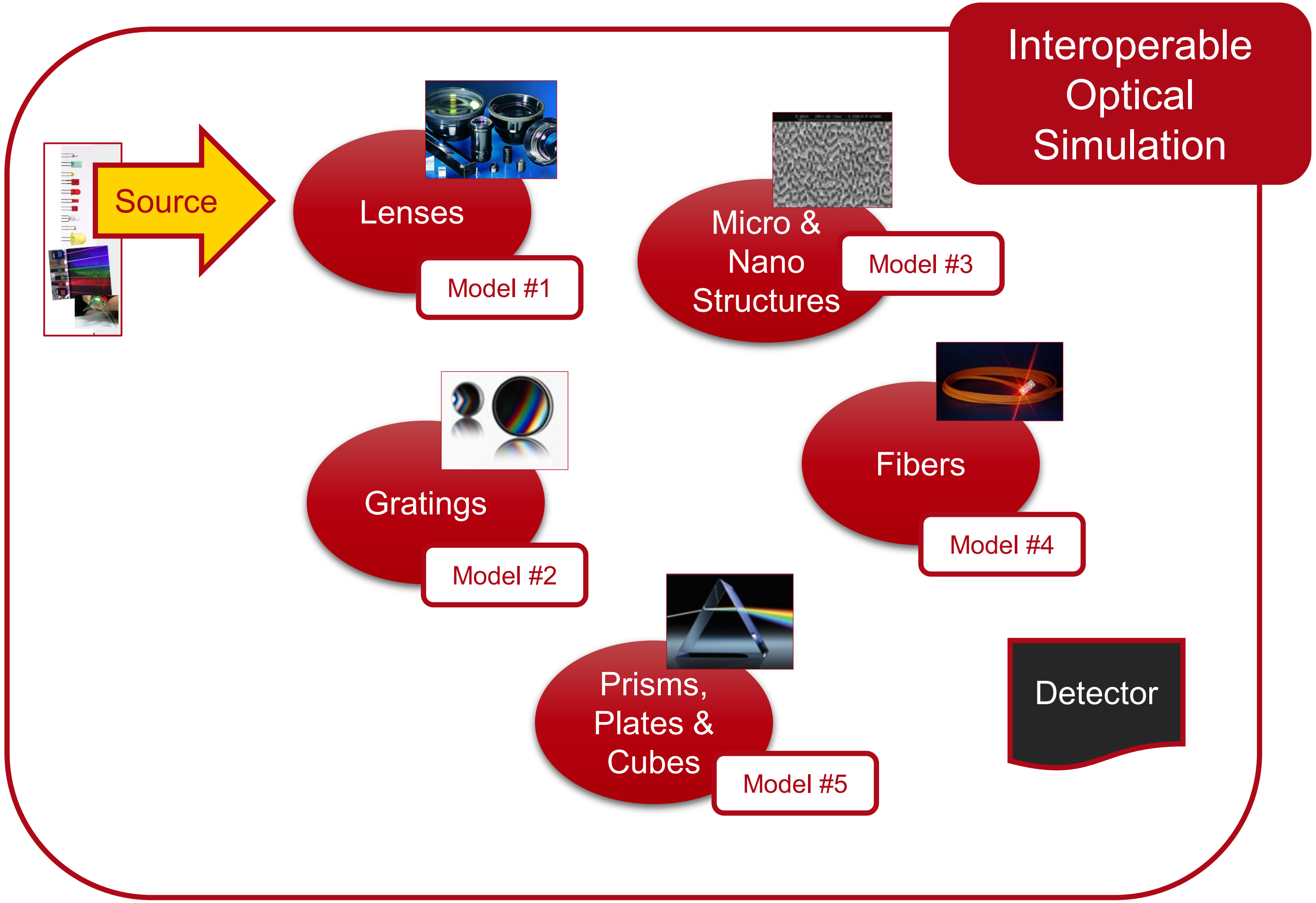

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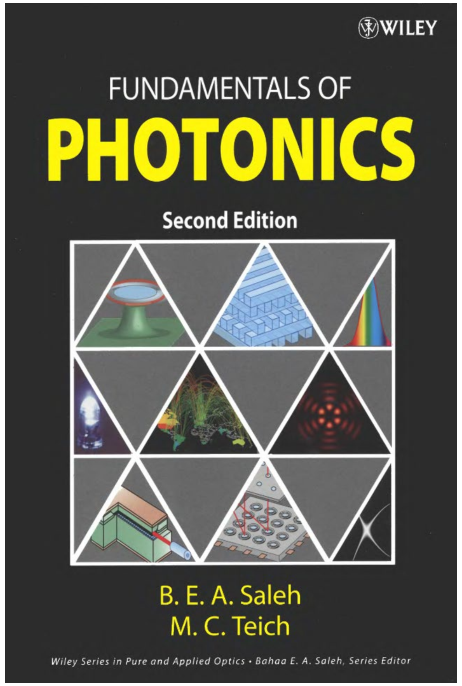
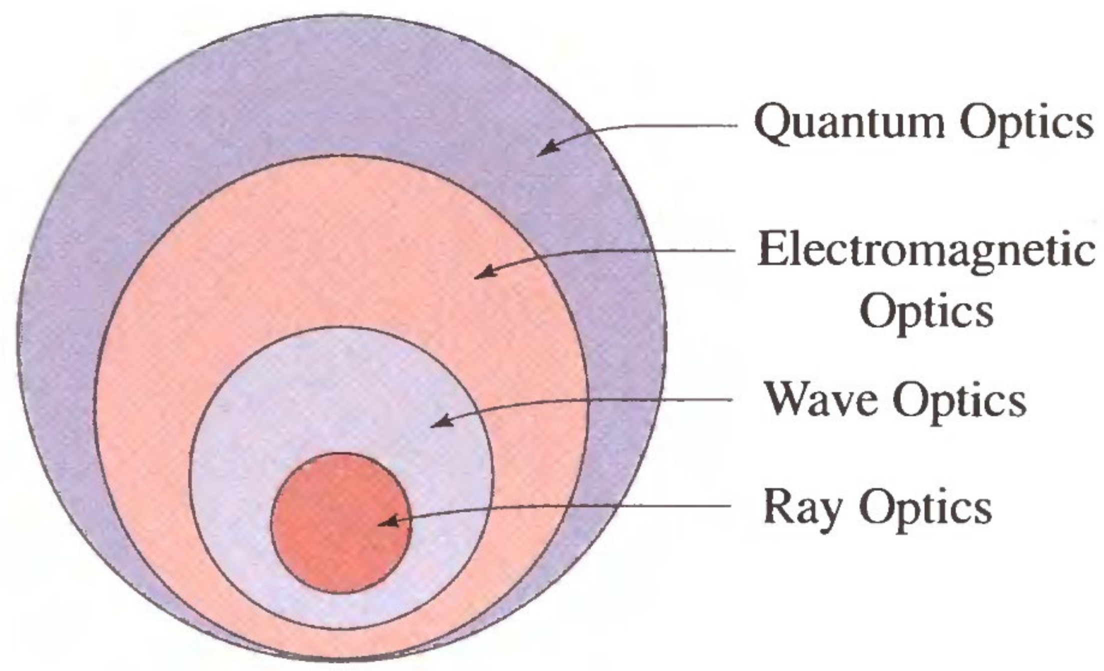
Interoperability?

Simulation Models and Light Representation





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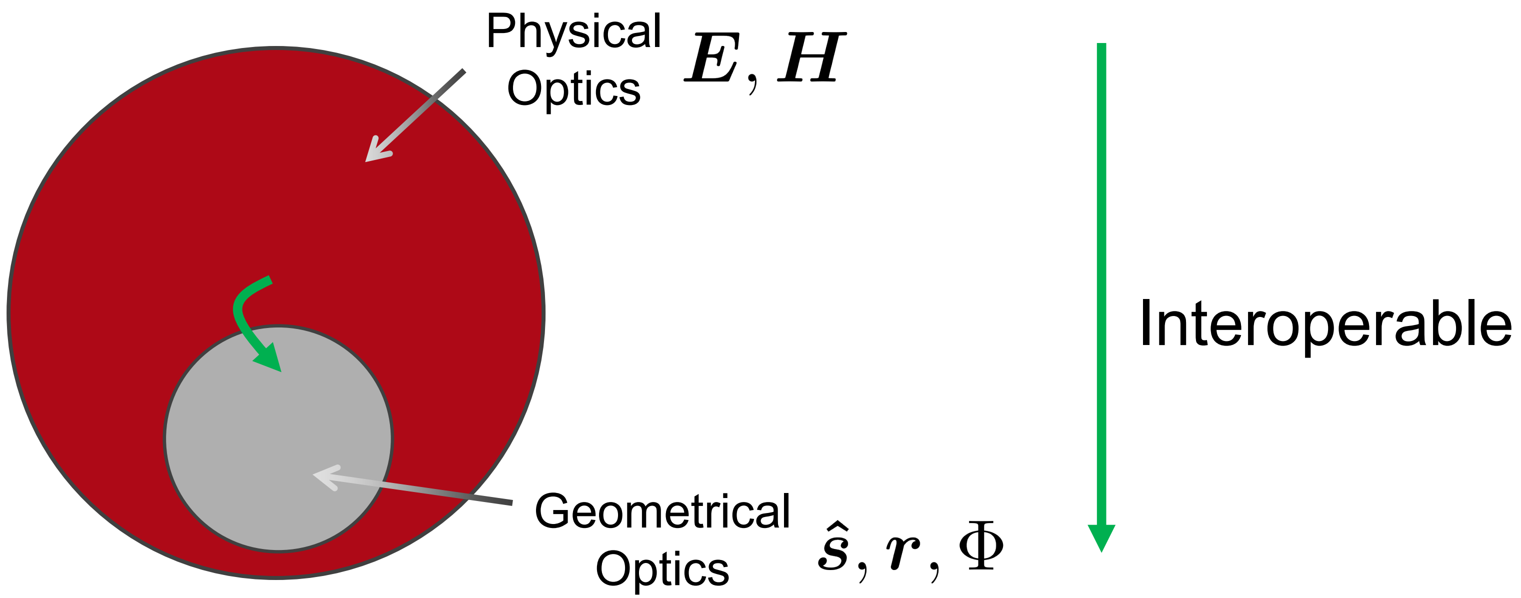

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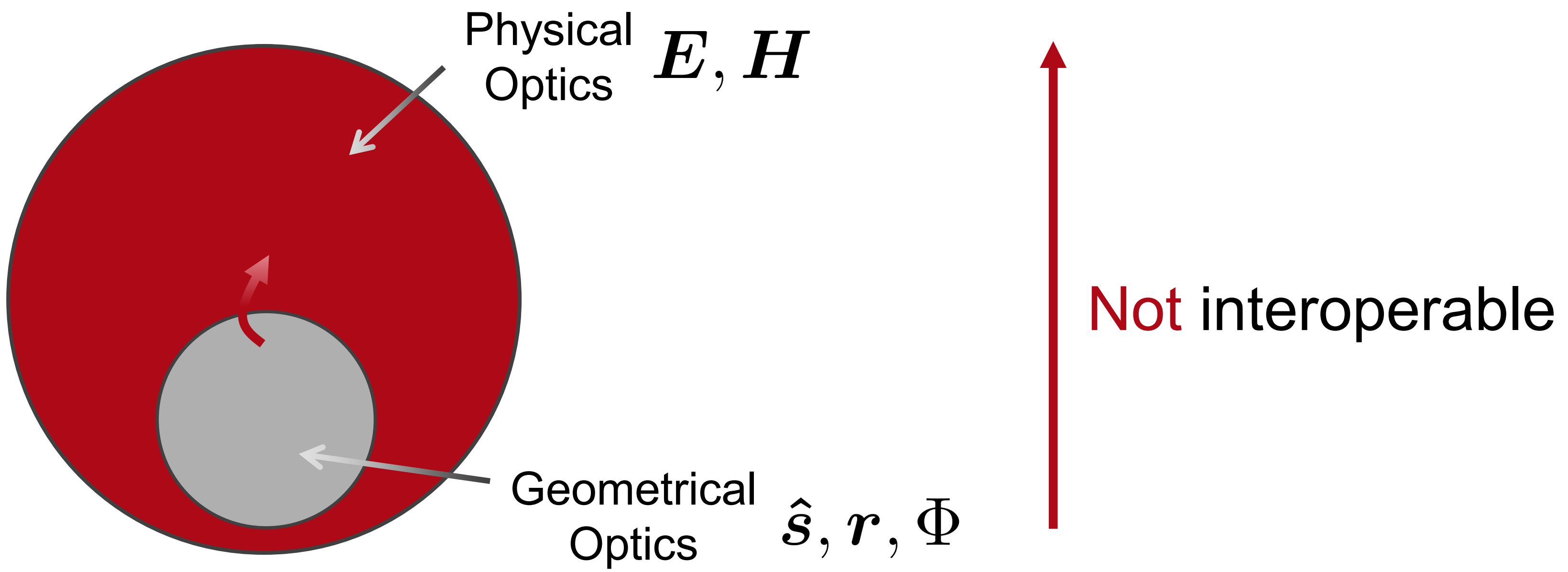
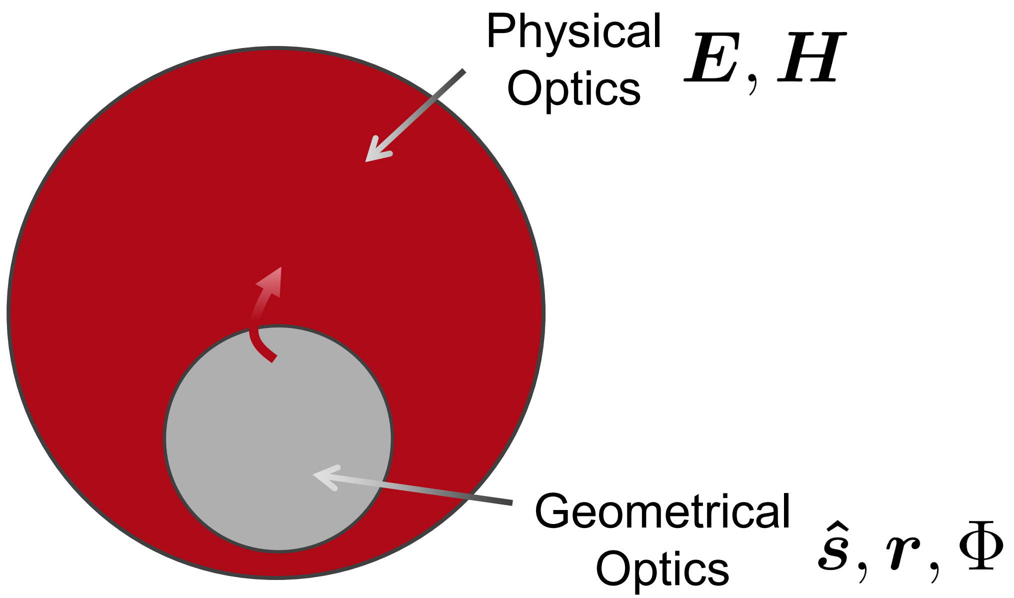


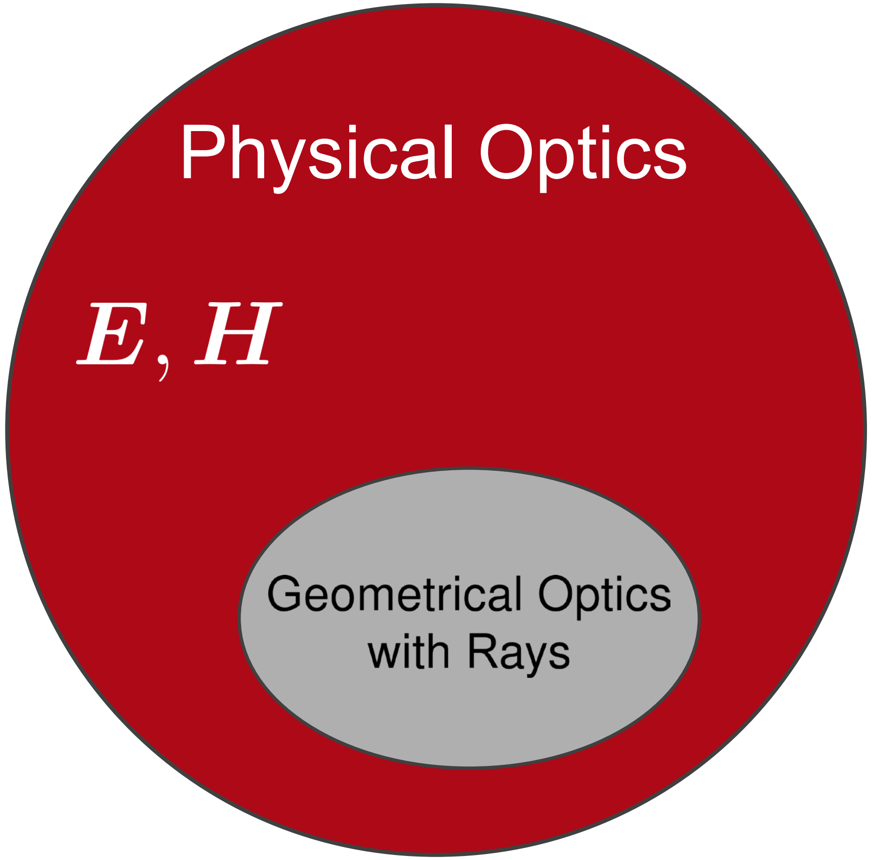
Interoperable Simulation Requires Generalized Geometrical Optics



Not interoperable

Geometrical optics for fields $\boldsymbol{E}, \boldsymbol{H}$ required!

Geometrical Optics for Electromagnetic Fields



Not interoperable

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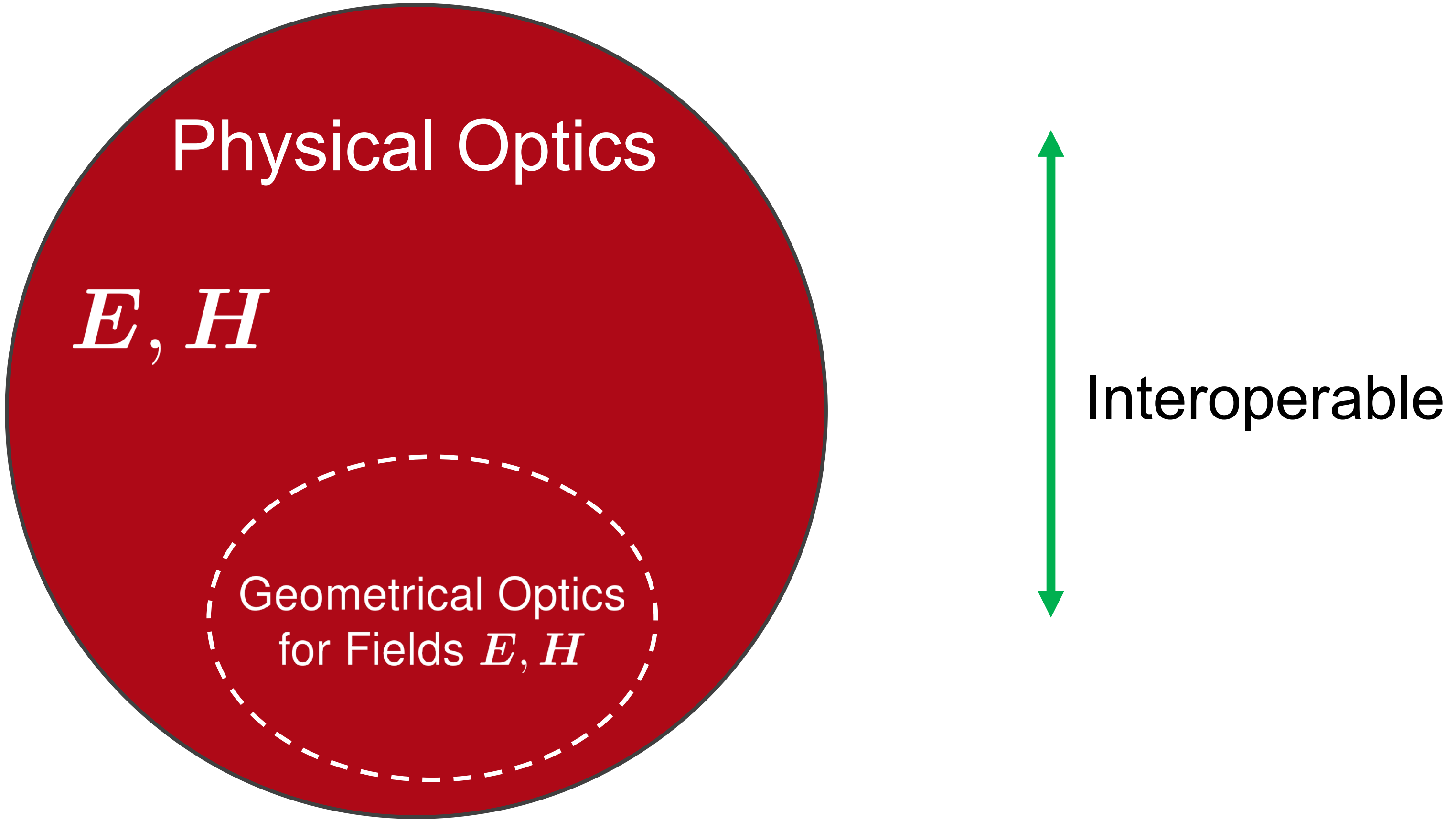

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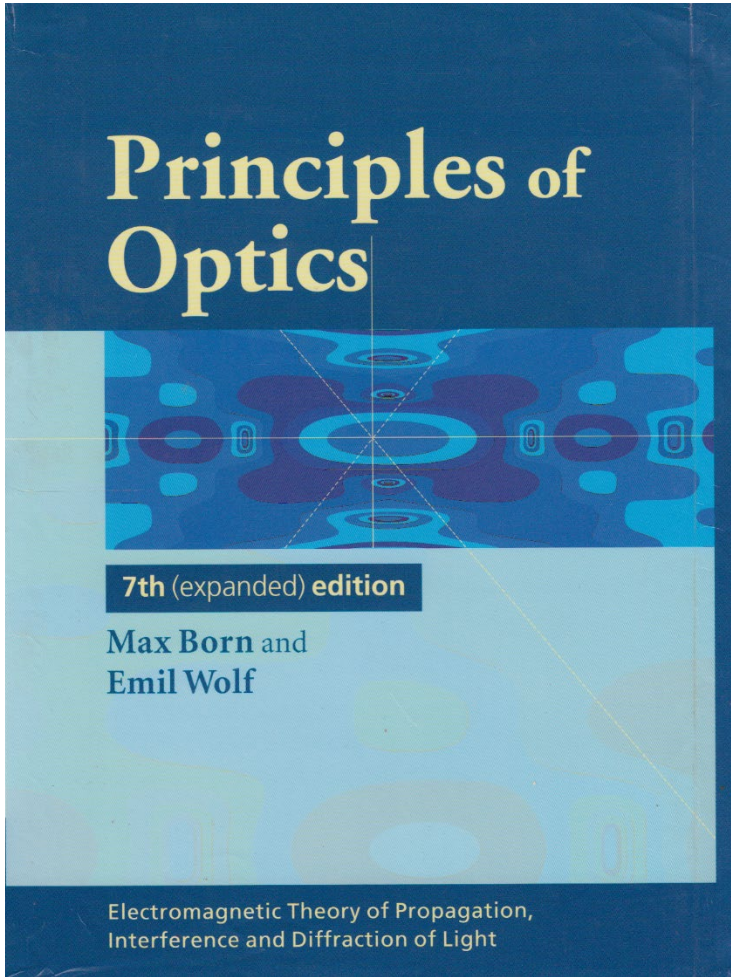

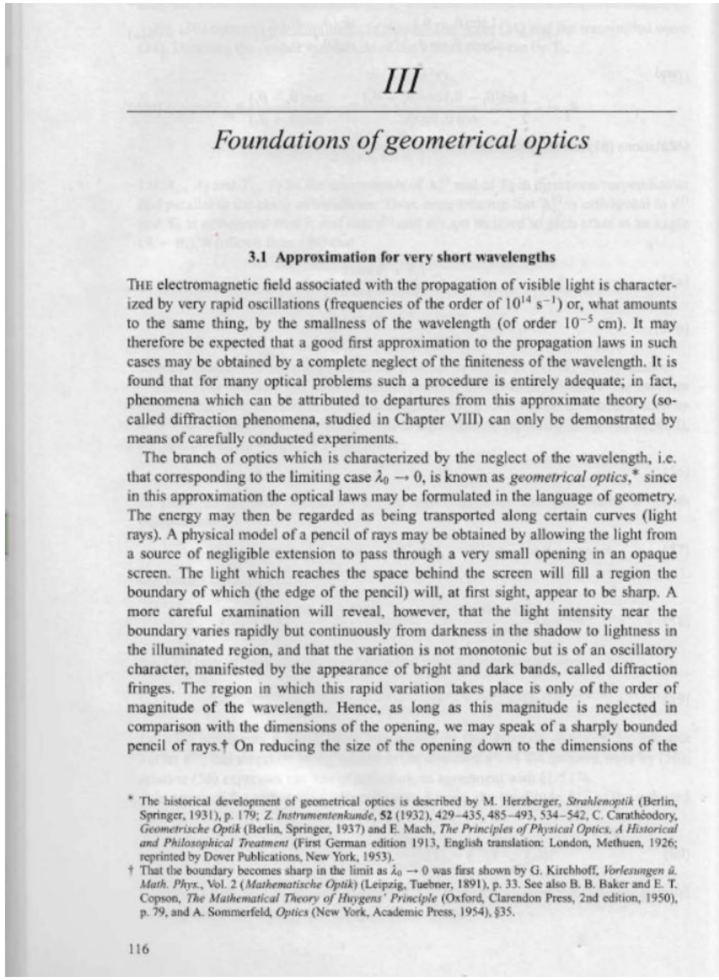

III

Foundations of geometrical optics

3.1 Approximation for very short wavelengths

THE electromagnetic field associated with the propagation of visible light is characterized by very rapid oscillations (frequencies of the order of $10^{14}\ s^{-1}$) or, what amounts to the same thing, by the smallness of the wavelength (of order 10^{-5} cm). It may therefore be expected that a good first approximation to the propagation laws in such cases may be obtained by a complete neglect of the finiteness of the wavelength. It is found that for many optical problems such a procedure is entirely adequate; in fact, phenomena which can be attributed to departures from this approximate theory (so-called diffraction phenomena, studied in Chapter VIII) can only be demonstrated by means of carefully conducted experiments.

The branch of optics which is characterized by the neglect of the wavelength, i.e. that corresponding to the limiting case $\lambda_0 \rightarrow 0$, is known as *geometrical optics*,* since in this approximation the optical laws may be formulated in the language of geometry. The energy may then be regarded as being transported along certain curves (light rays). A physical model of a pencil of rays may be obtained by allowing the light from a source of negligible extension to pass through a very small opening in an opaque screen. The light which reaches the space behind the screen will fill a region the boundary of which (the edge of the pencil) will, at first sight, appear to be sharp. A more careful examination will reveal, however, that the light intensity near the boundary varies rapidly but continuously from darkness in the shadow to lightness in the illuminated region, and that the variation is not monotonic but is of an oscillatory character, manifested by the appearance of bright and dark bands, called diffraction fringes. The region in which this rapid variation takes place is only of the order of magnitude of the wavelength. Hence, as long as this magnitude is neglected in comparison with the dimensions of the opening, we may speak of a sharply bounded pencil of rays.† On reducing the size of the opening down to the dimensions of the

* The historical development of geometrical optics is described by M. Herzberger, *Strahlenoptik* (Berlin, Springer, 1931), p. 179; *Z. Instrumentenkunde*, **52** (1932), 429–435, 485–493, 534–542, C. Carathéodory, *Geometrische Optik* (Berlin, Springer, 1937) and E. Mach, *The Principles of Physical Optics, A Historical and Philosophical Treatment* (First German edition 1913, English translation: London, Methuen, 1926; reprinted by Dover Publications, New York, 1953).

† That the boundary becomes sharp in the limit as $\lambda_0 \rightarrow 0$ was first shown by G. Kirchhoff, *Vorlesungen ü. Math. Phys.*, Vol. 2 (*Mathematische Optik*) (Leipzig, Tuebner, 1891), p. 33. See also B. B. Baker and E. T. Copson, *The Mathematical Theory of Huygens' Principle* (Oxford, Clarendon Press, 2nd edition, 1950), p. 79, and A. Sommerfeld, *Optics* (New York, Academic Press, 1954), §35.

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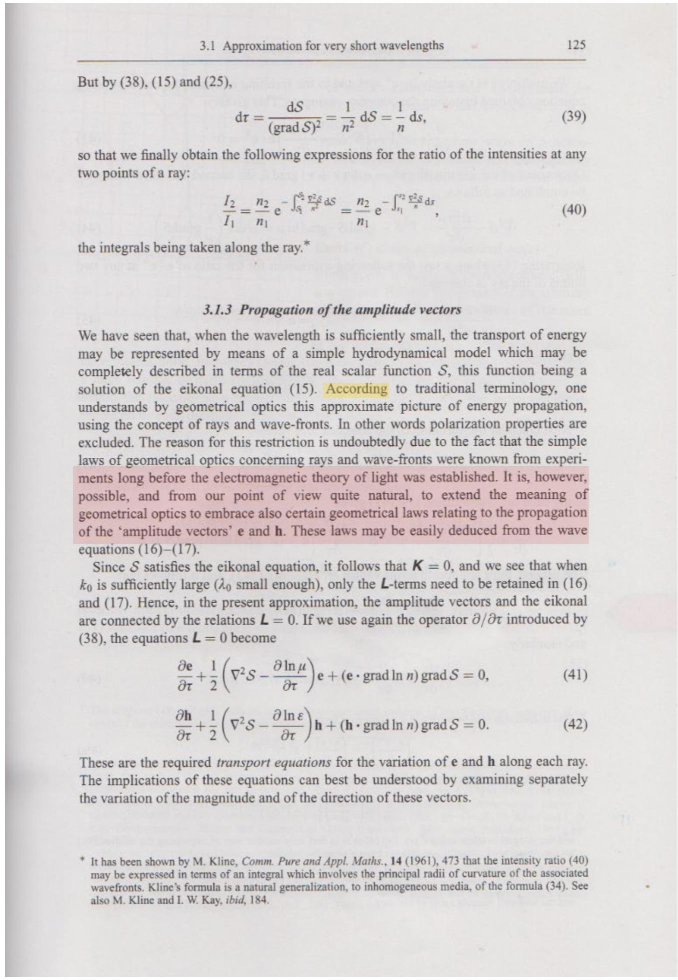
Geometrical Optics for Electromagnetic Fields


3.1 Approximation for very short wavelengths 125

But by (38), (15) and (25),

$$d\tau = \frac{d\mathcal{S}}{(\text{grad}\,\mathcal{S})^2} = \frac{1}{n^2}\,d\mathcal{S} = \frac{1}{n}\,ds, \tag{39}$$

so that we finally obtain the following expressions for the ratio of the intensities at any two points of a ray:

$$\frac{I_2}{I_1} = \frac{n_2}{n_1}\,e^{-\int_{\mathcal{S}_1}^{\mathcal{S}_2}\frac{\nabla^2\mathcal{S}}{n^2}\,d\mathcal{S}} = \frac{n_2}{n_1}\,e^{-\int_{s_1}^{s_2}\frac{\nabla^2\mathcal{S}}{n}\,ds}, \tag{40}$$

the integrals being taken along the ray.*

3.1.3 *Propagation of the amplitude vectors*

We have seen that, when the wavelength is sufficiently small, the transport of energy may be represented by means of a simple hydrodynamical model which may be completely described in terms of the real scalar function $\mathcal{S}$, this function being a solution of the eikonal equation (15). According to traditional terminology, one understands by geometrical optics this approximate picture of energy propagation, using the concept of rays and wave-fronts. In other words polarization properties are excluded. The reason for this restriction is undoubtedly due to the fact that the simple laws of geometrical optics concerning rays and wave-fronts were known from experiments long before the electromagnetic theory of light was established. It is, however, possible, and from our point of view quite natural, to extend the meaning of geometrical optics to embrace also certain geometrical laws relating to the propagation of the 'amplitude vectors' **e** and **h**. These laws may be easily deduced from the wave equations (16)–(17).

Since $\mathcal{S}$ satisfies the eikonal equation, it follows that $\boldsymbol{K} = 0$, and we see that when k_0 is sufficiently large (λ_0 small enough), only the $\boldsymbol{L}$-terms need to be retained in (16) and (17). Hence, in the present approximation, the amplitude vectors and the eikonal are connected by the relations $\boldsymbol{L} = 0$. If we use again the operator $\partial/\partial\tau$ introduced by (38), the equations $\boldsymbol{L} = 0$ become

$$\frac{\partial \mathbf{e}}{\partial \tau} + \frac{1}{2}\left(\nabla^2\mathcal{S} - \frac{\partial \ln \mu}{\partial \tau}\right)\mathbf{e} + (\mathbf{e}\cdot\text{grad}\ln n)\,\text{grad}\,\mathcal{S} = 0, \tag{41}$$

$$\frac{\partial \mathbf{h}}{\partial \tau} + \frac{1}{2}\left(\nabla^2\mathcal{S} - \frac{\partial \ln \varepsilon}{\partial \tau}\right)\mathbf{h} + (\mathbf{h}\cdot\text{grad}\ln n)\,\text{grad}\,\mathcal{S} = 0. \tag{42}$$

These are the required *transport equations* for the variation of **e** and **h** along each ray. The implications of these equations can best be understood by examining separately the variation of the magnitude and of the direction of these vectors.

* It has been shown by M. Kline, *Comm. Pure and Appl. Maths.*, **14** (1961), 473 that the intensity ratio (40) may be expressed in terms of an integral which involves the principal radii of curvature of the associated wavefronts. Kline's formula is a natural generalization, to inhomogeneous media, of the formula (34). See also M. Kline and I. W. Kay, *ibid*, 184.

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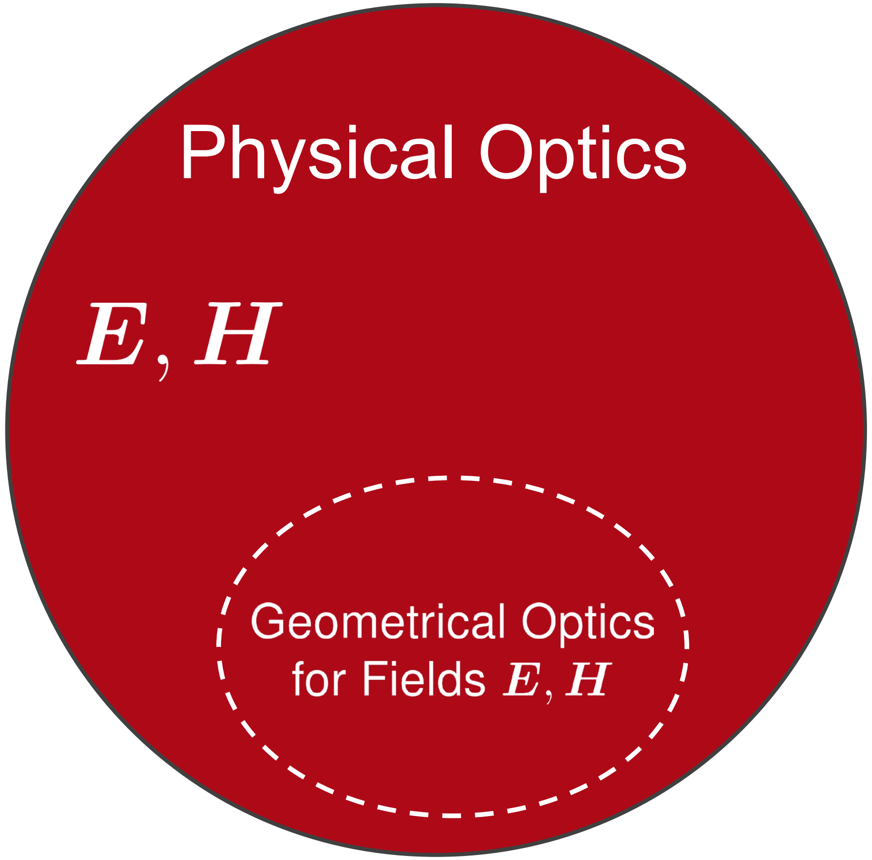

We follow Max Born's and Emil Wolf's advice!

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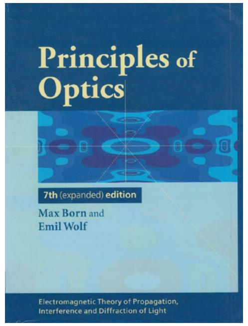
Citation from 

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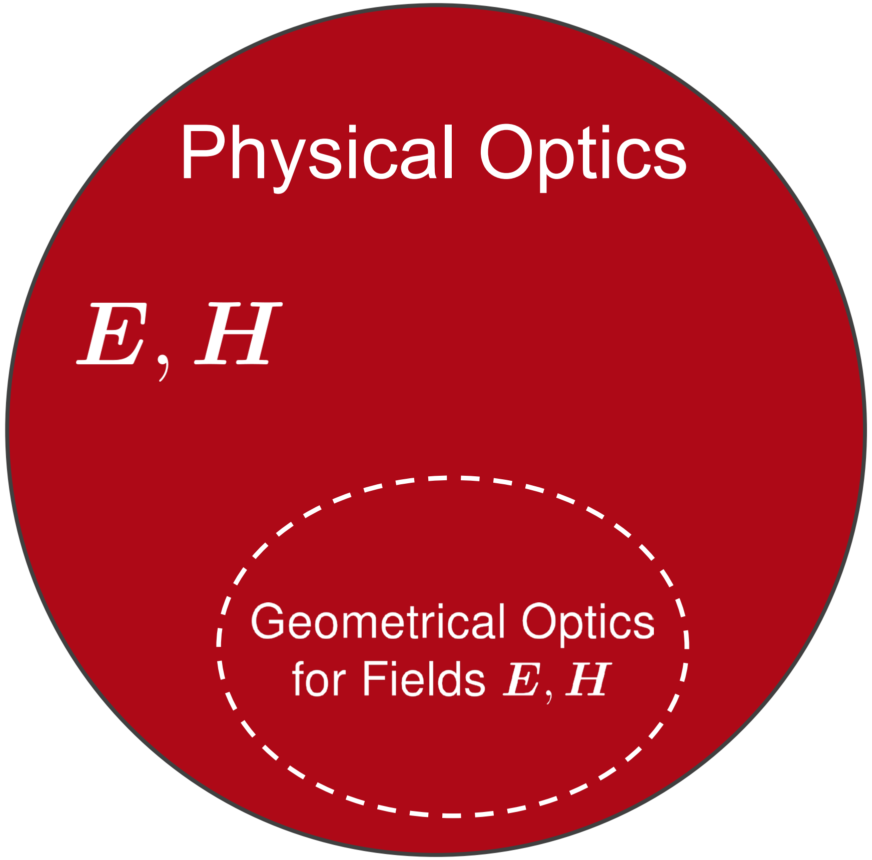

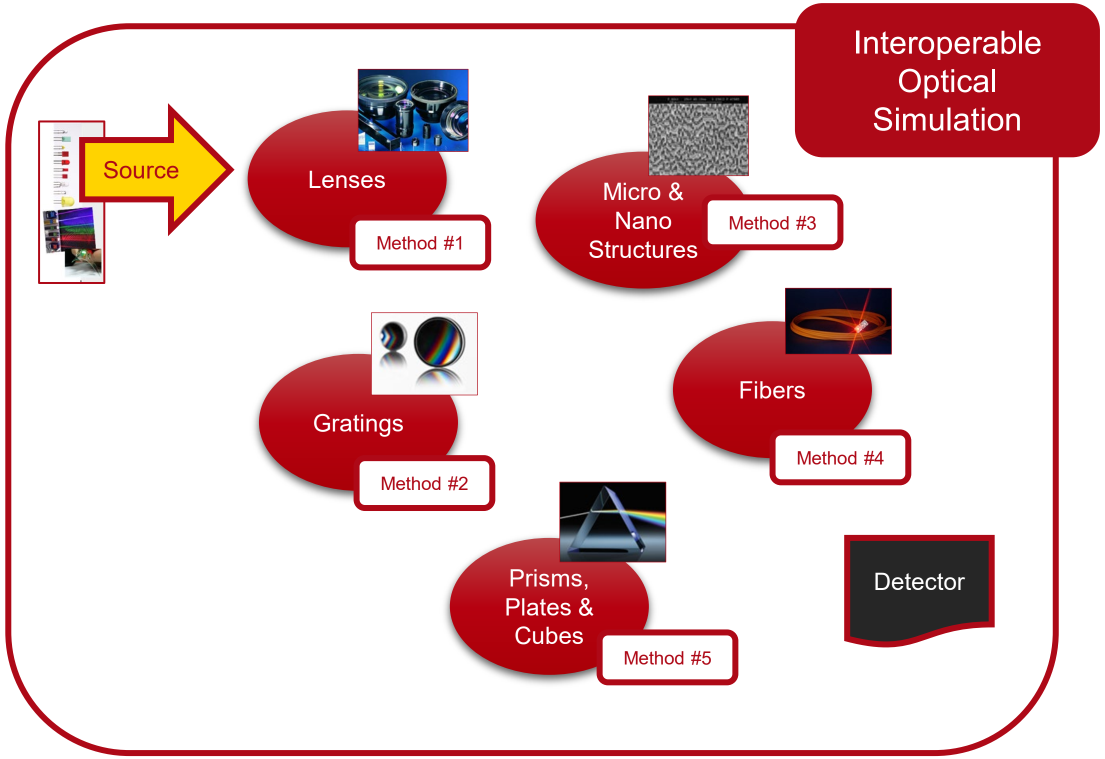

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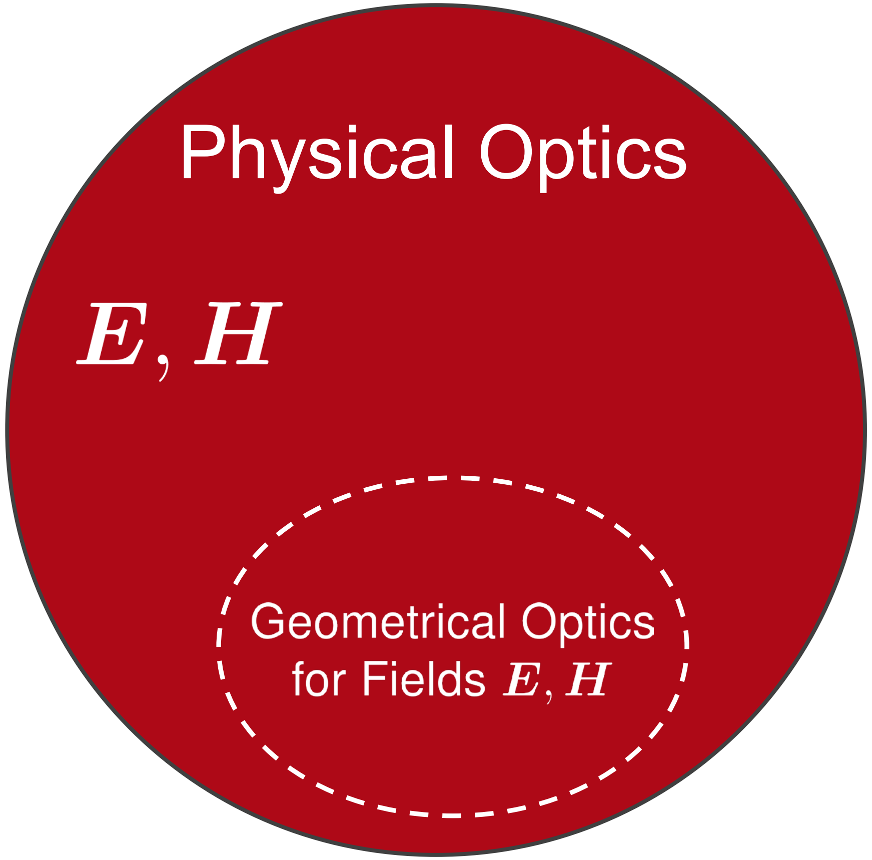

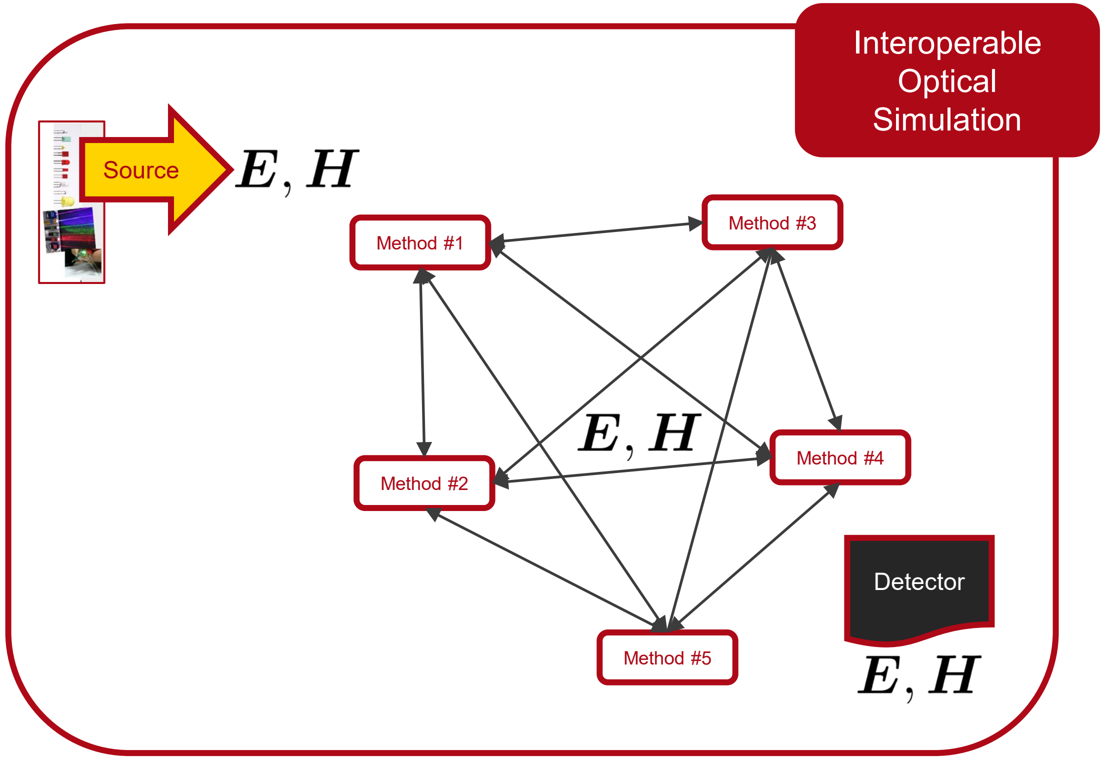

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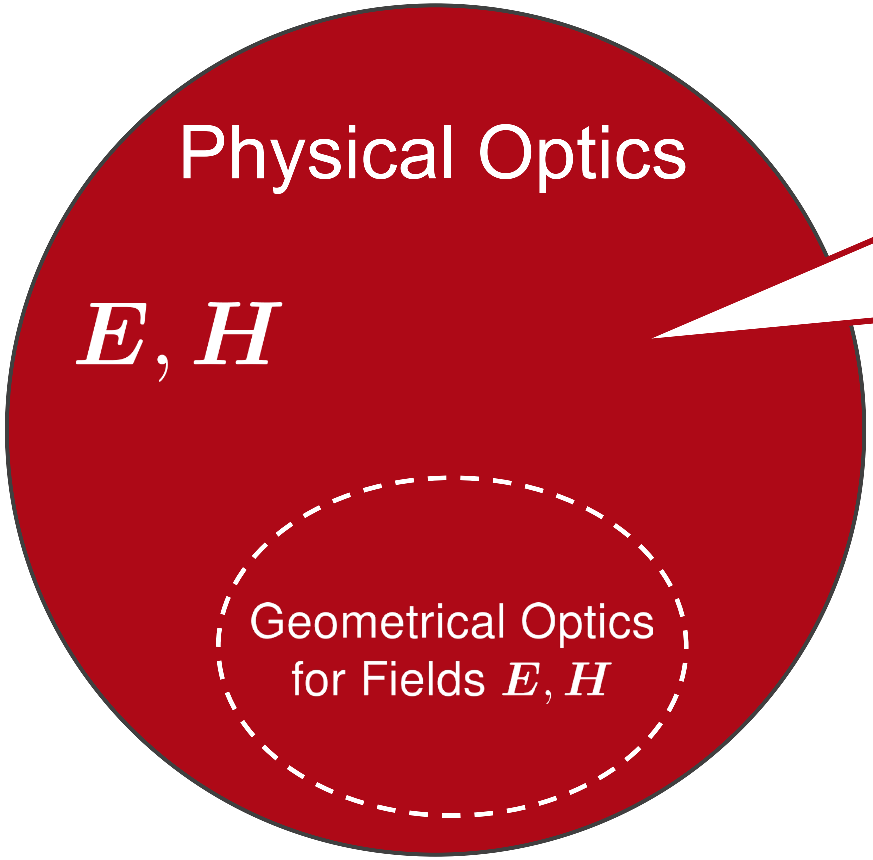

Modeling of optical effects, including, e.g., aberrations, energy redistribution, diffraction, scattering, interference, speckles, polarization, coherence, and spatiotemporal evolution.

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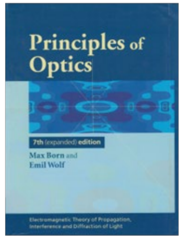
Citation from  page 125

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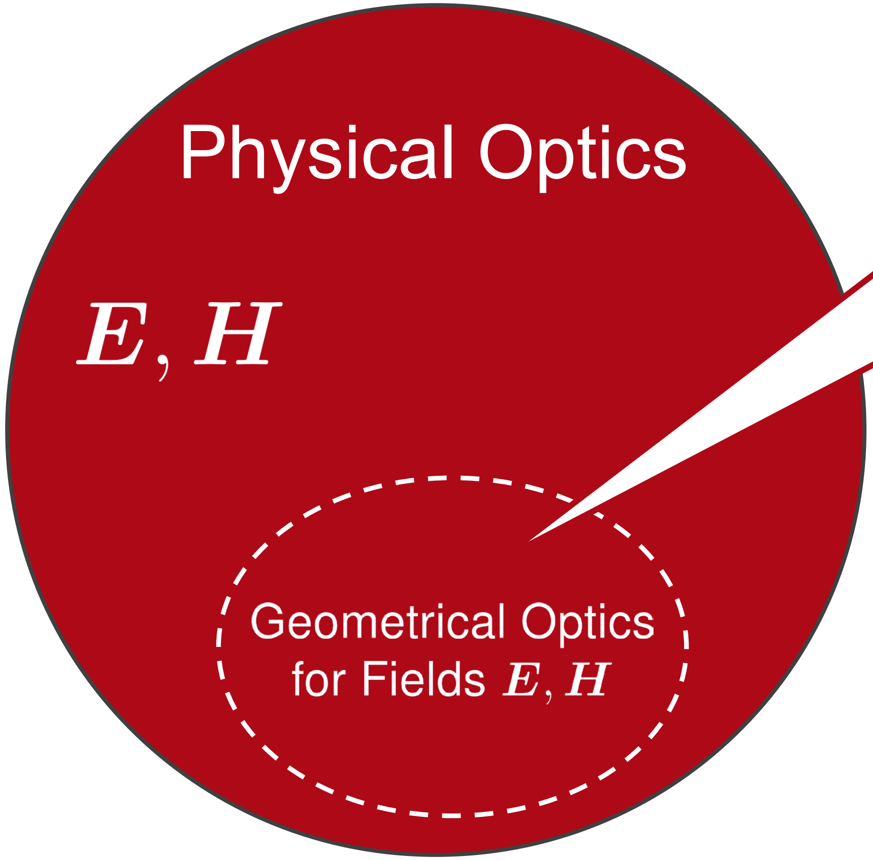

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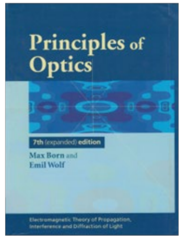

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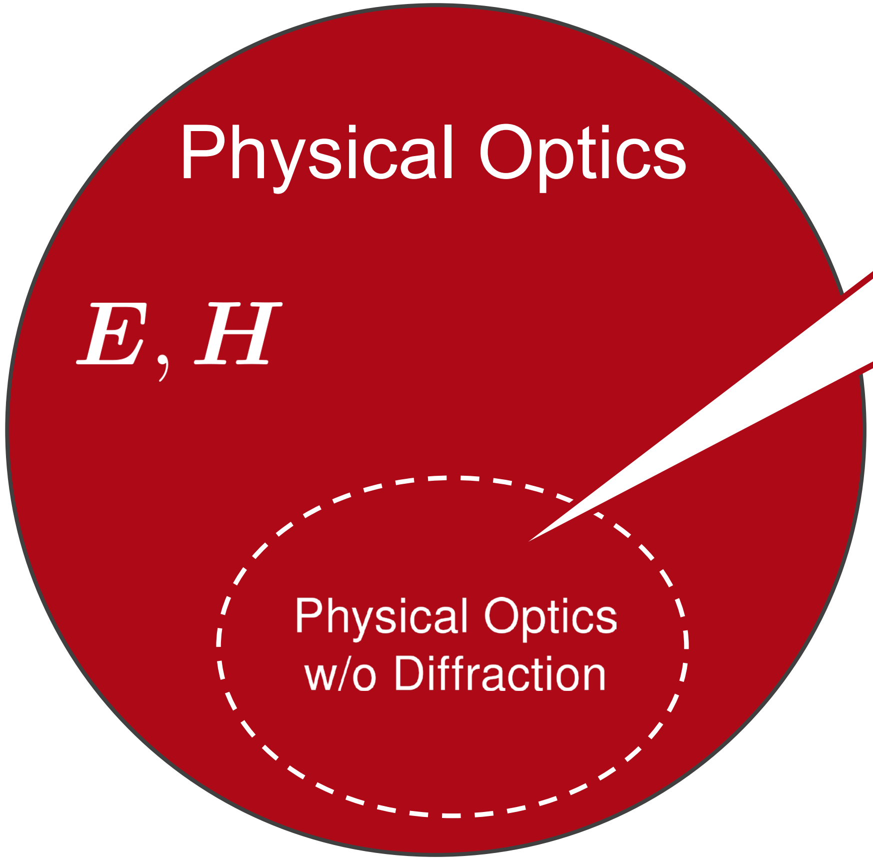

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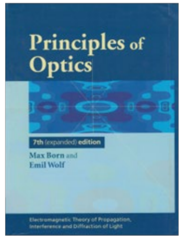
Citation from  page 125

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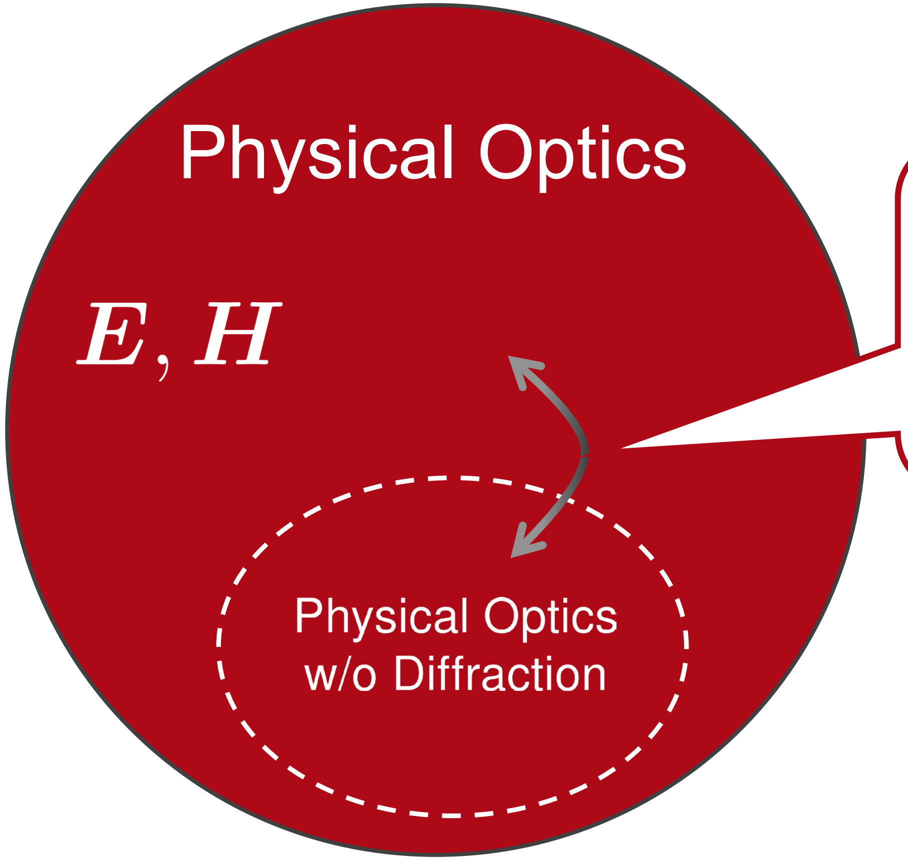
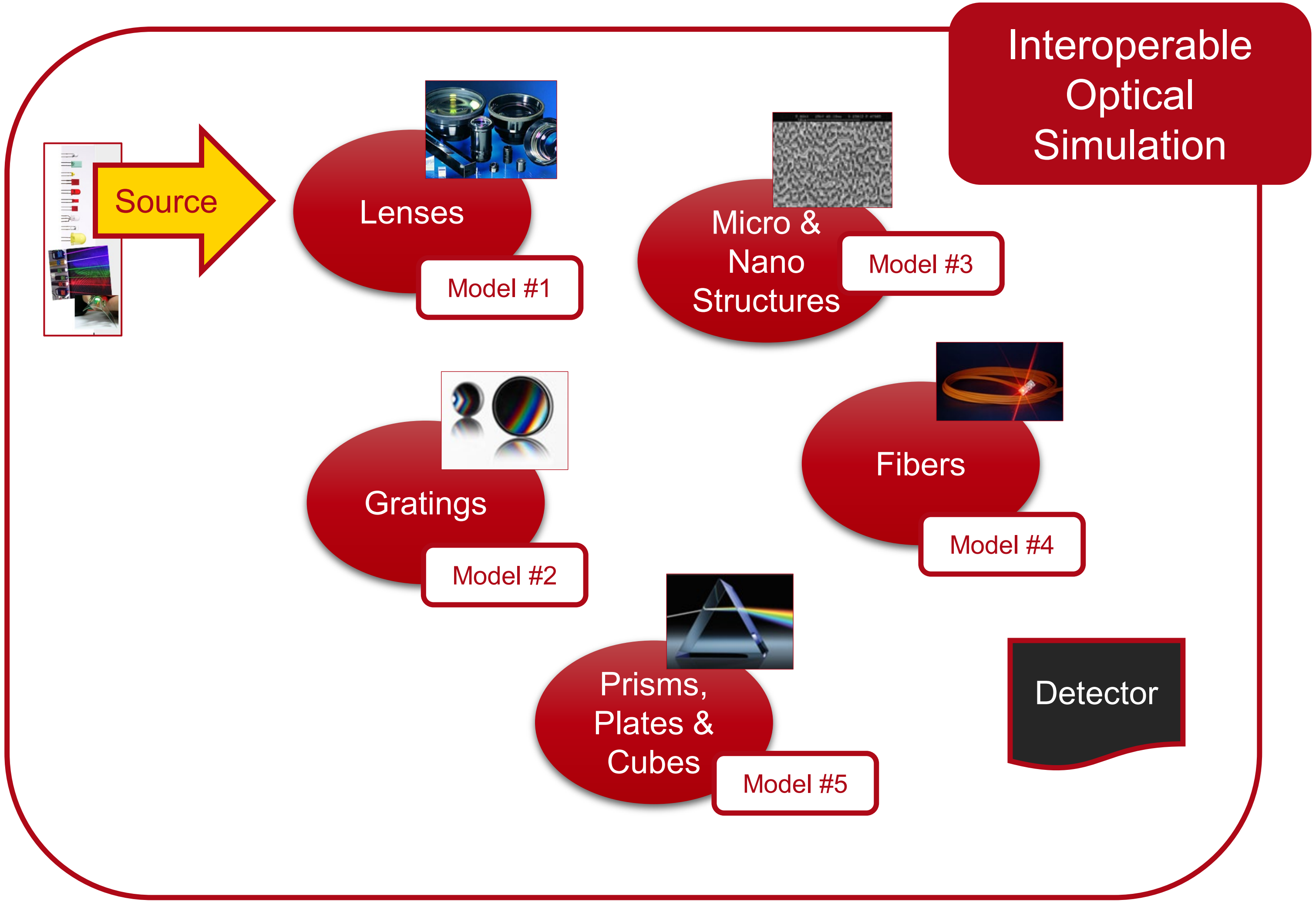


Enables the effective control of the extent of diffraction effects taken into account in simulations using physical optics.

Simulating Optical Systems: Combine Many Techniques



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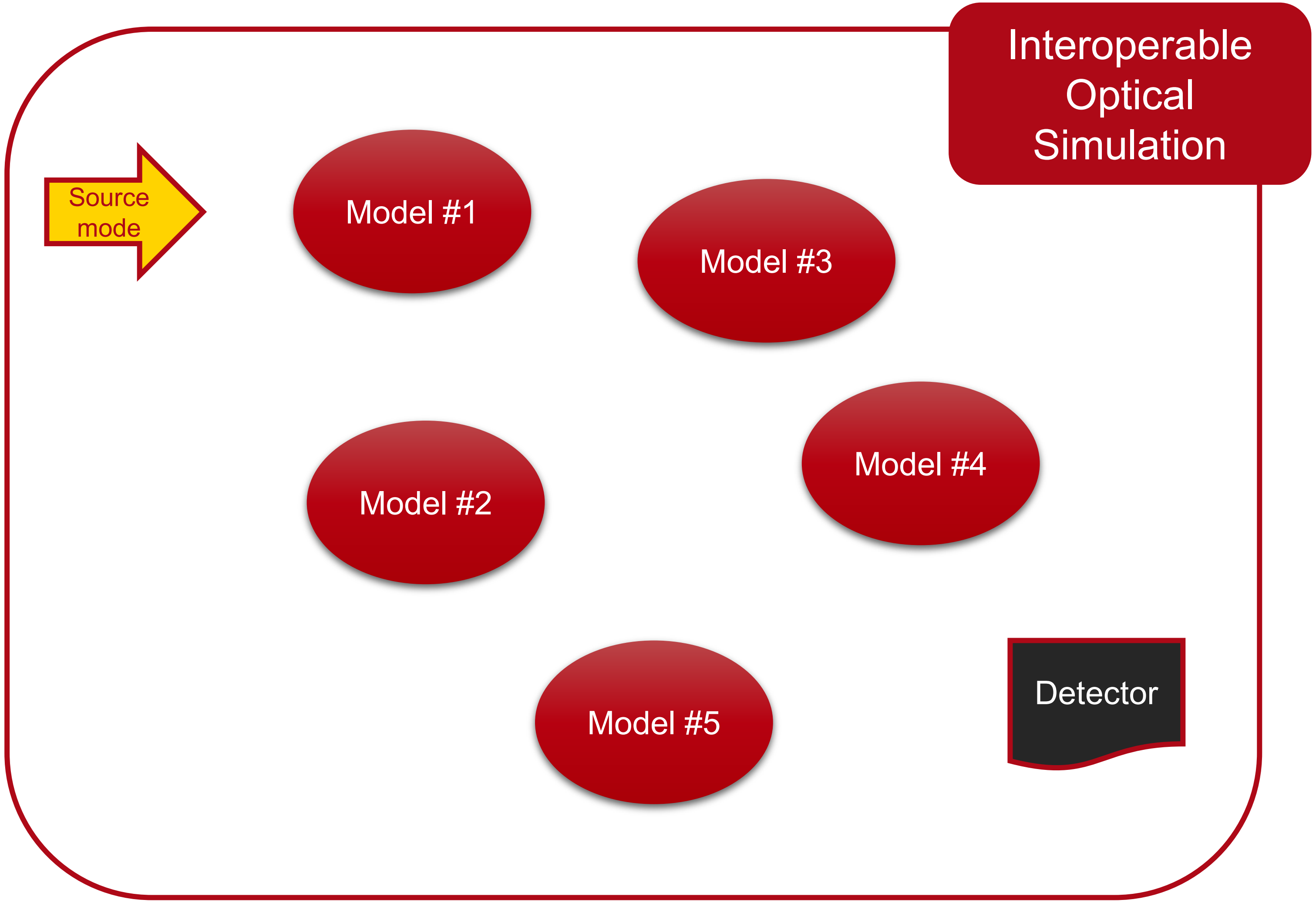
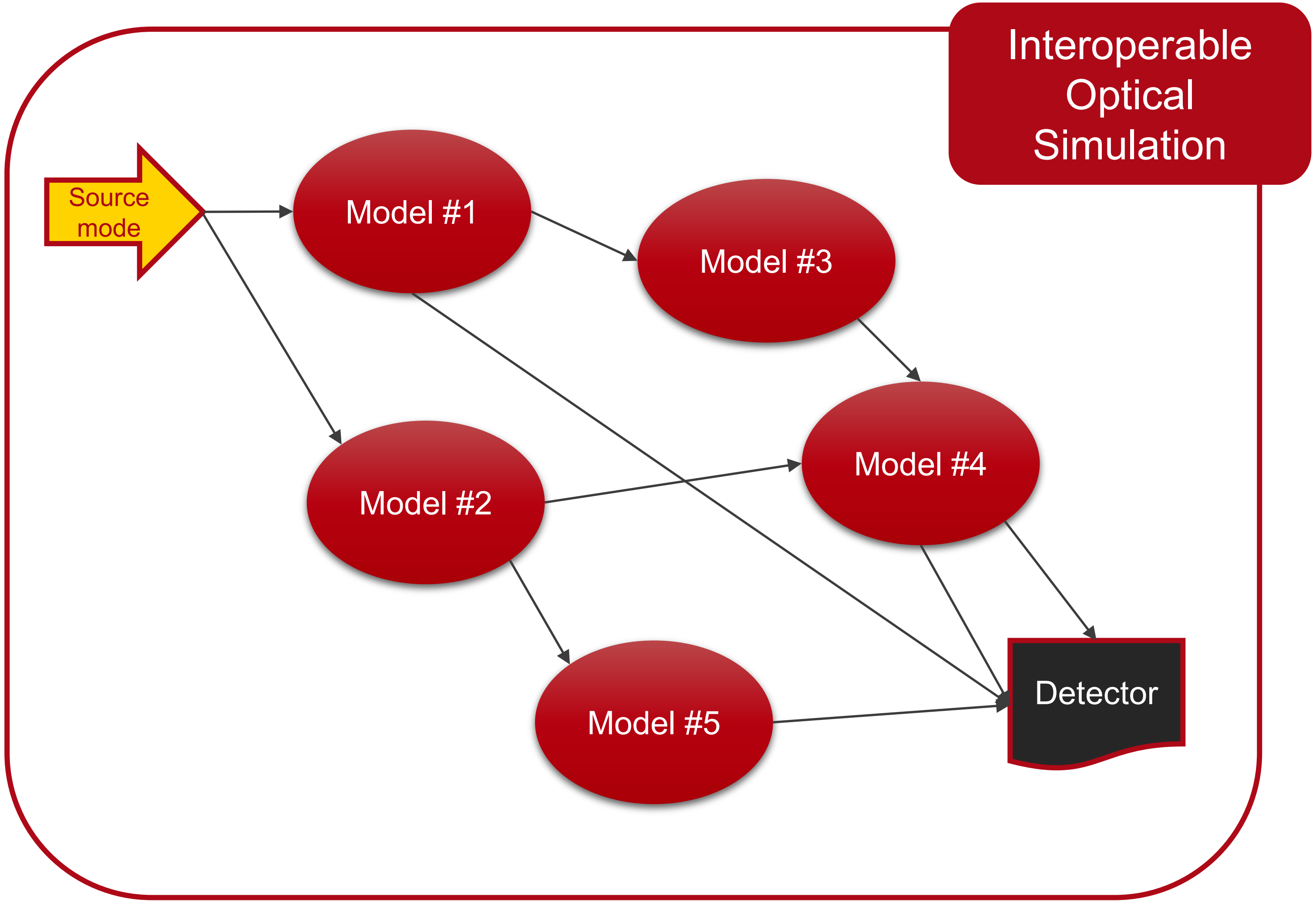
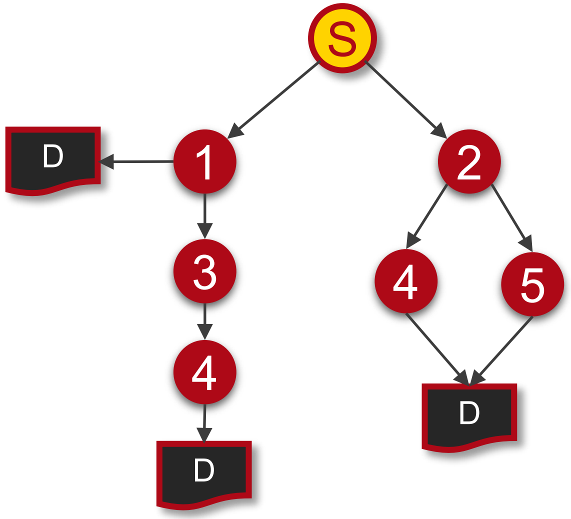
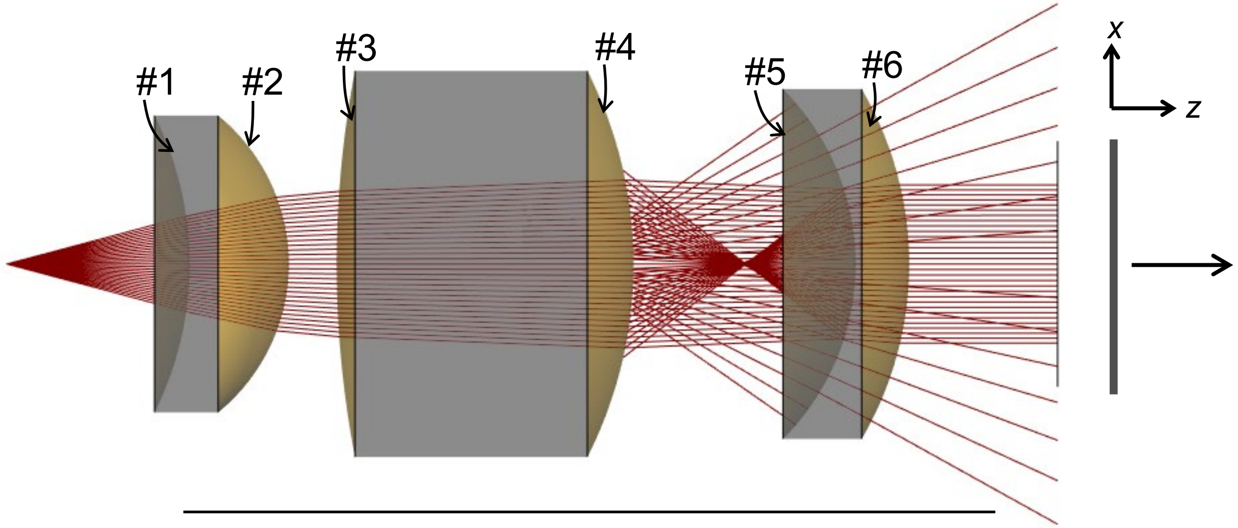


Interoperable Optical Simulation is Non-Sequential

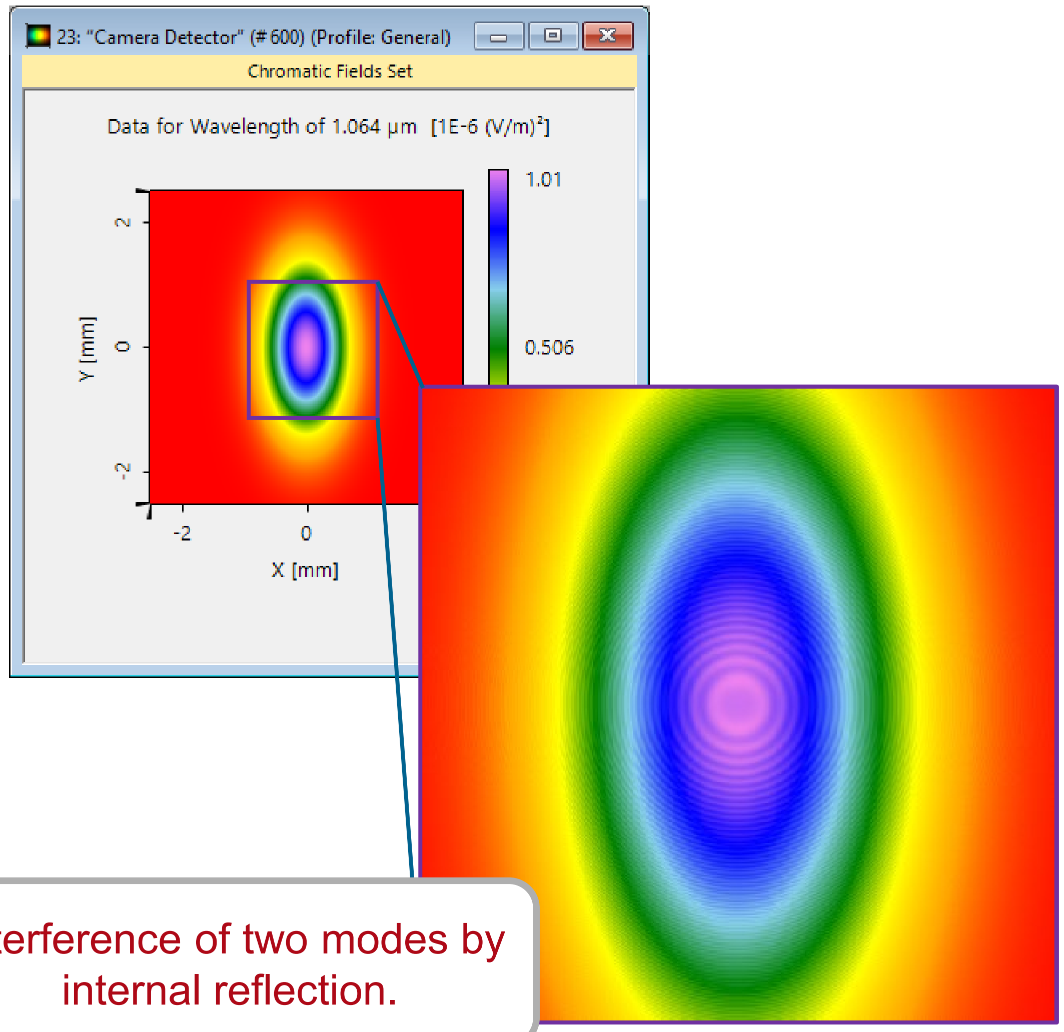




Scenario: Lens System Modeling with Ghost Signal

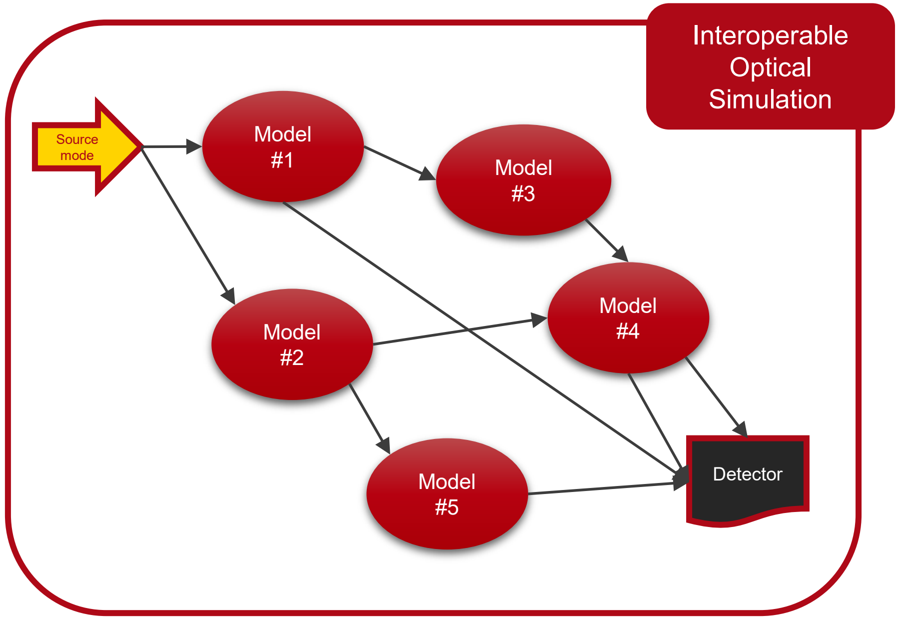


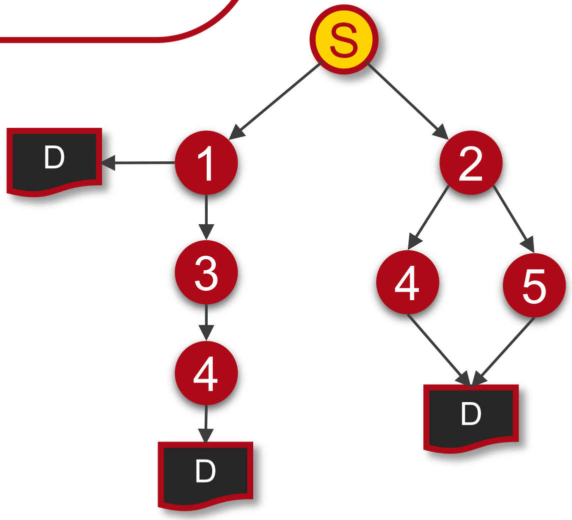
	#1	#2	#3	#4	#5	#6
+/+	√	√	√	√	√	√
+/-					√	
-/-						
-/+				√		



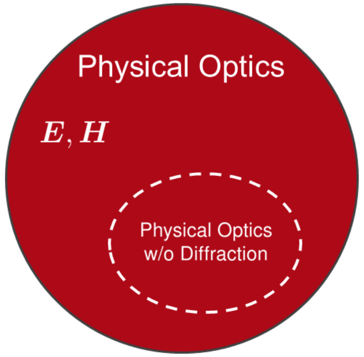
Interference of two modes by internal reflection.

Interoperable Optical Simulation is Non-Sequential





- The nodes of the simulation tree represent the simulation models for each component.
- The connections among the nodes illustrate the propagation of light between the components.
- **Diffraction manifests itself during this propagation.**



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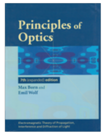
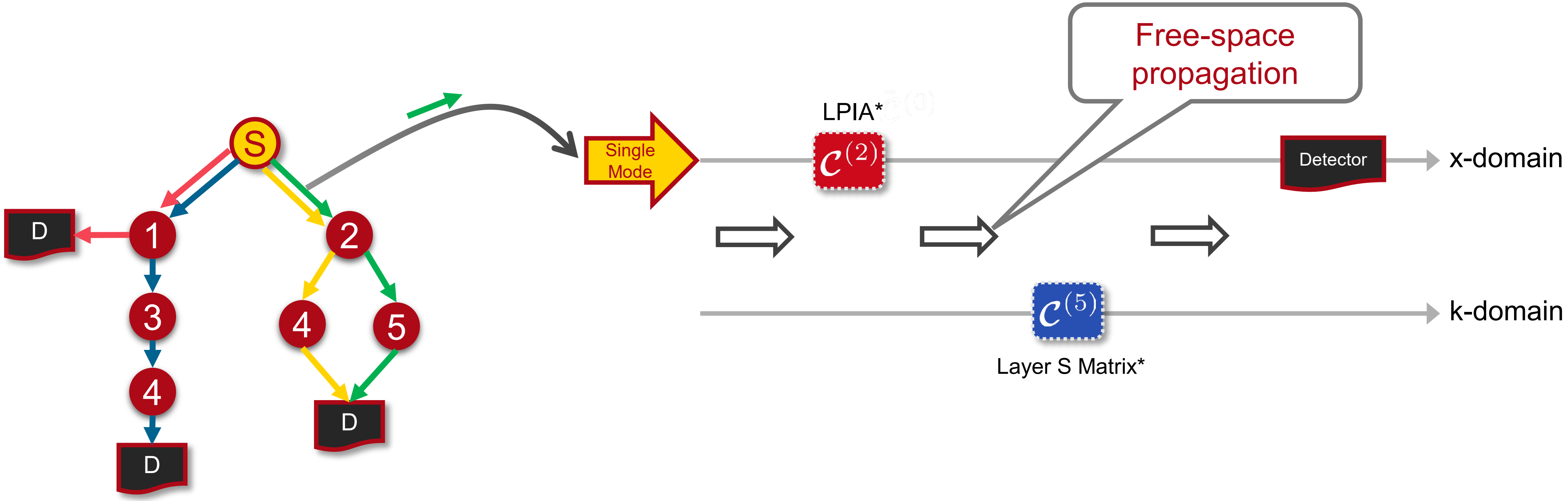


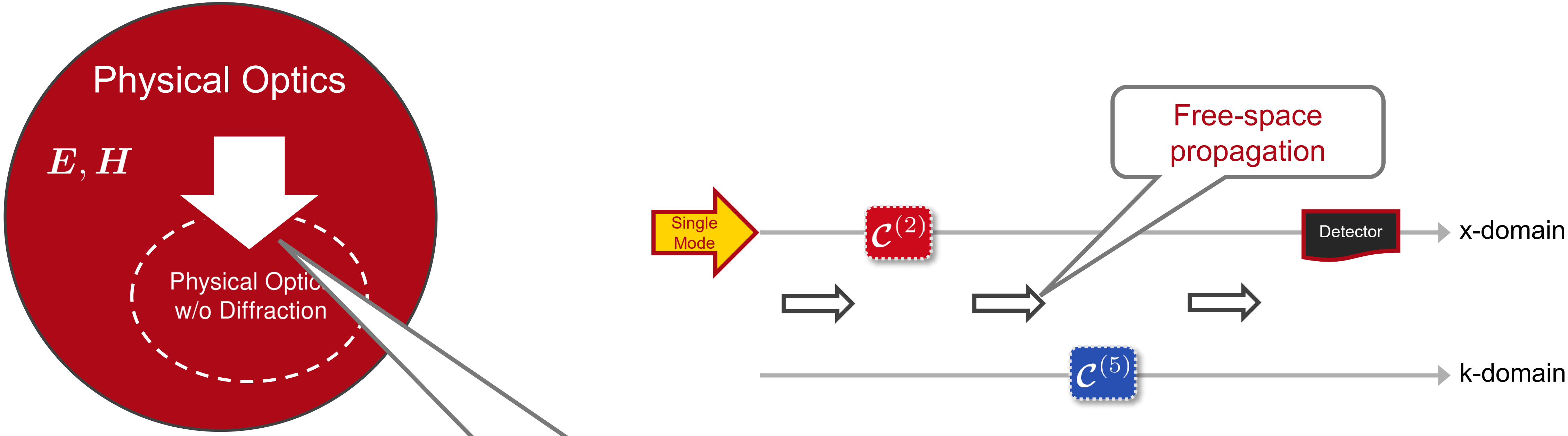
From Simulation Tree to Operator Sequences

- The simulation tree can be divided into a limited number of **operator sequences**.
- These sequences can be illustrated using a modeling diagram.



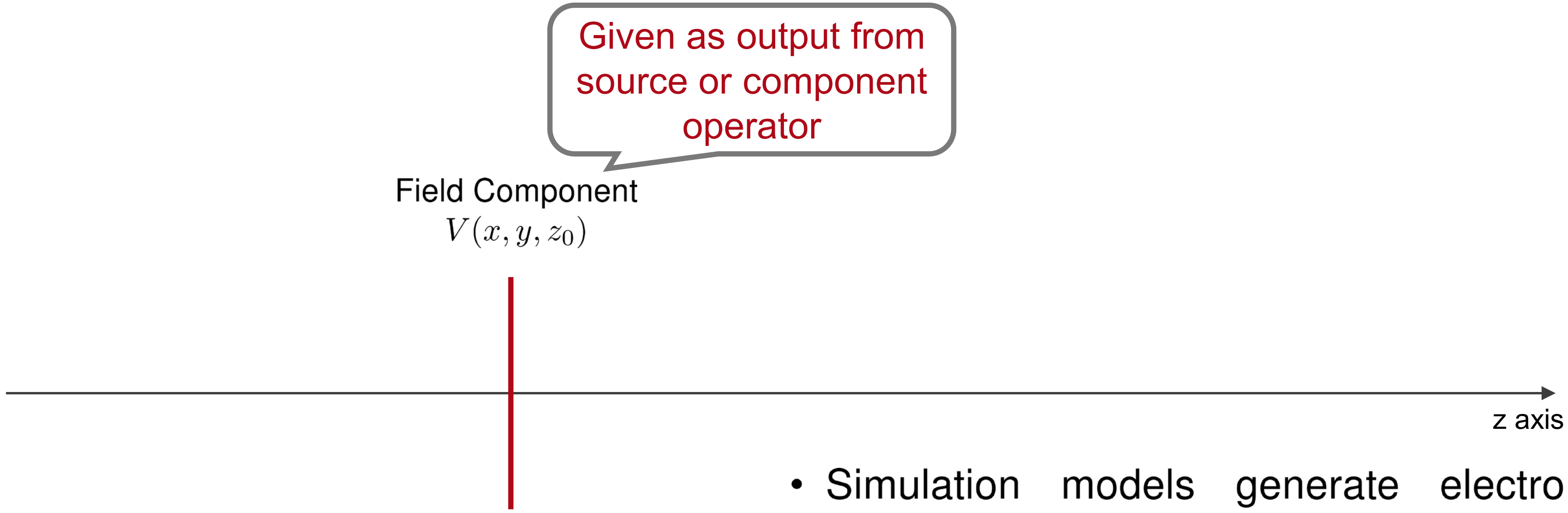
* Here we assume a curved and a planar surface as an example.

Modeling Free-Space Propagation



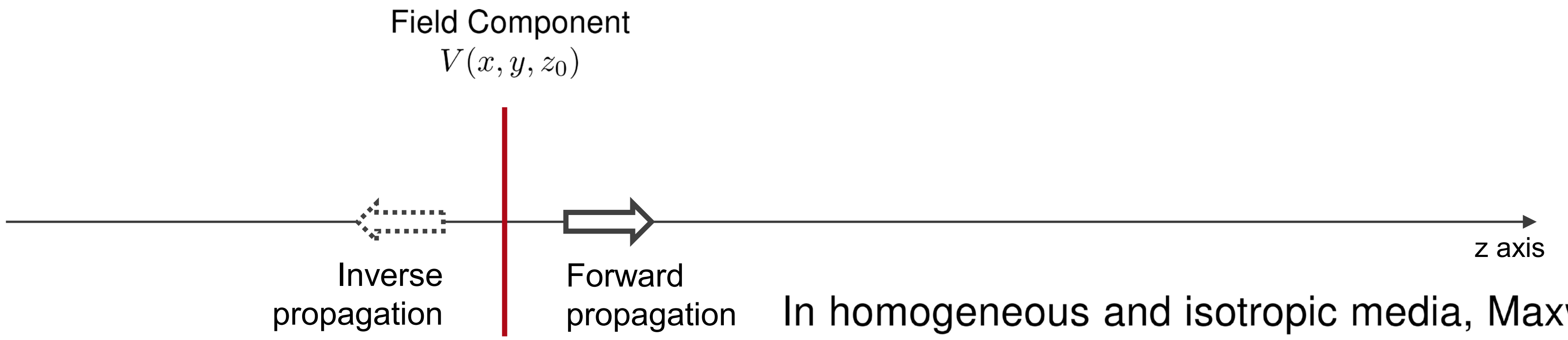
Derive generalized geometrical optics in the **framework of physical optics**.

Electromagnetic Fields in Homogeneous, Isotropic Media



- Simulation models generate electromagnetic fields $\boldsymbol{E}$ and $\boldsymbol{H}$ at the output planes of sources and components.
- These fields are then propagated to following components and detectors in the sequence.

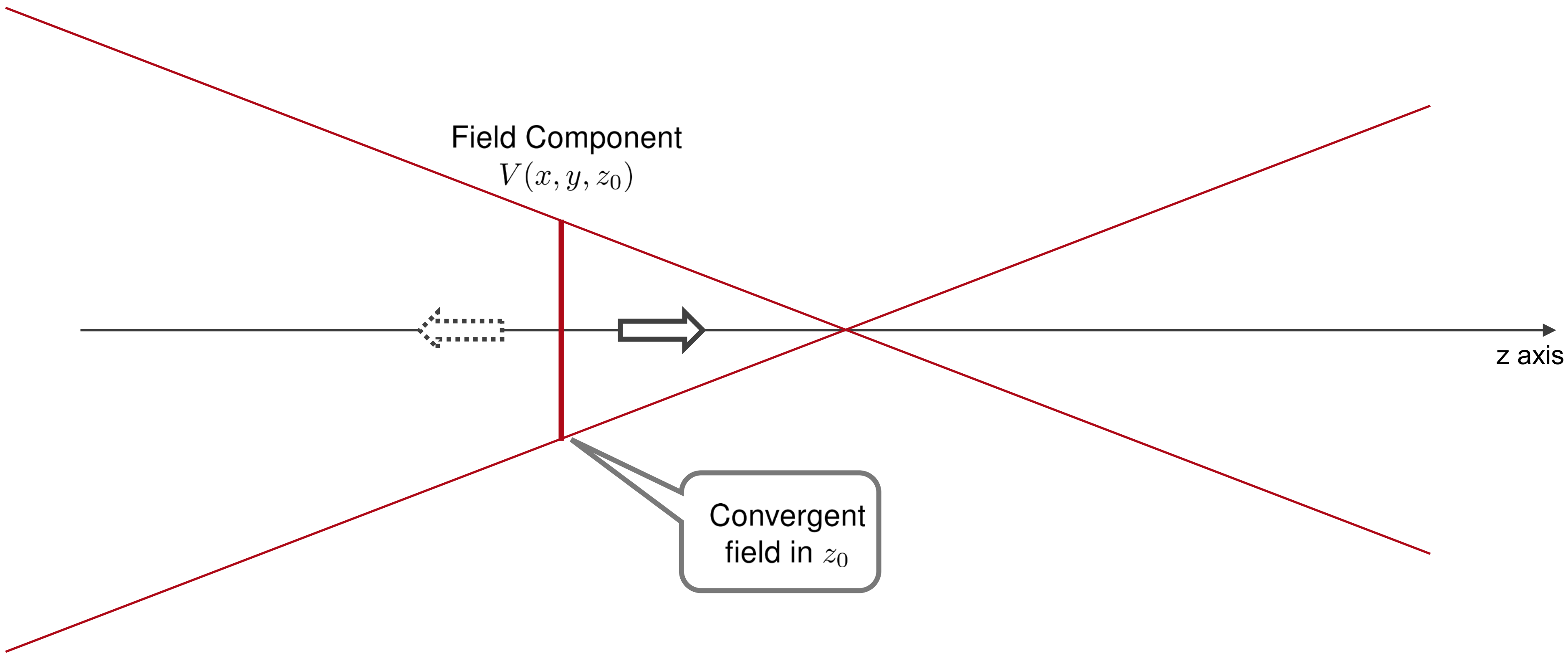
Electromagnetic Fields in Homogeneous, Isotropic Media



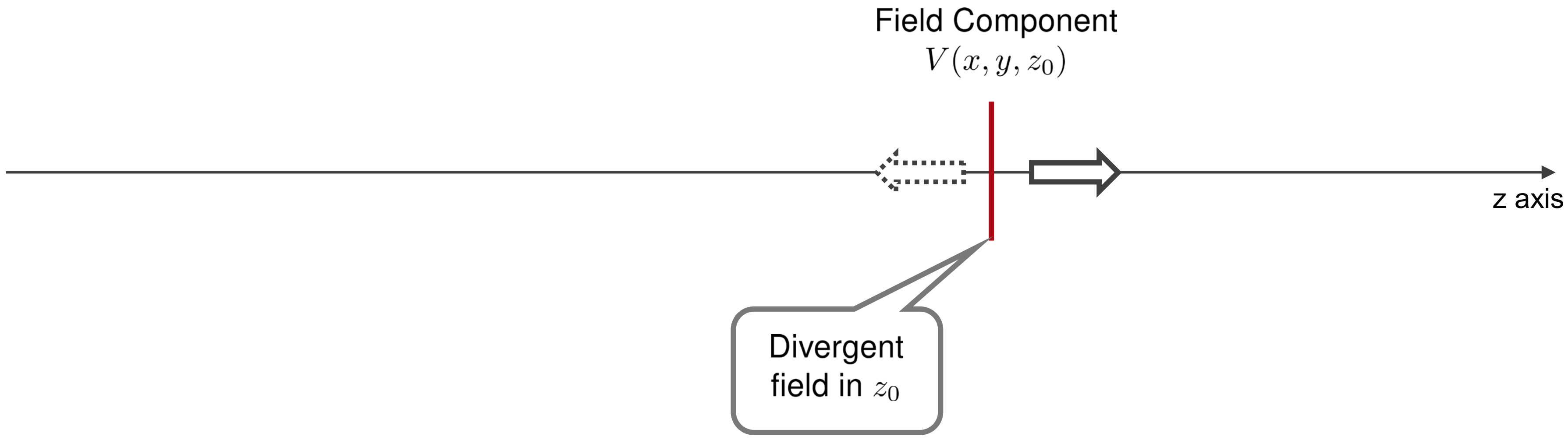
In homogeneous and isotropic media, Maxwell's equations state that:

- By utilizing $\boldsymbol{E}(x, y, z_0)$ and $\boldsymbol{H}(x, y, z_0)$, we can derive $\boldsymbol{E}(x, y, z)$ and $\boldsymbol{H}(x, y, z)$ for $z > z_0$ through forward propagation and for $z < z_0$ through inverse propagation.
- The propagation of each field component, denoted by V, can be carried out independently.

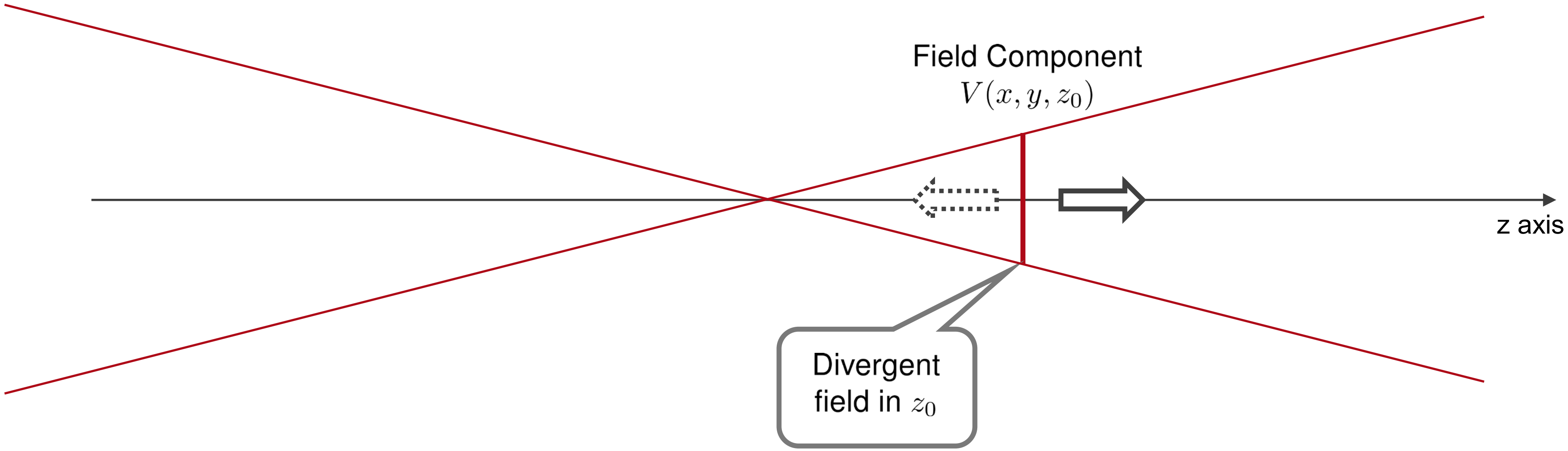
Electromagnetic Fields in Homogeneous, Isotropic Media



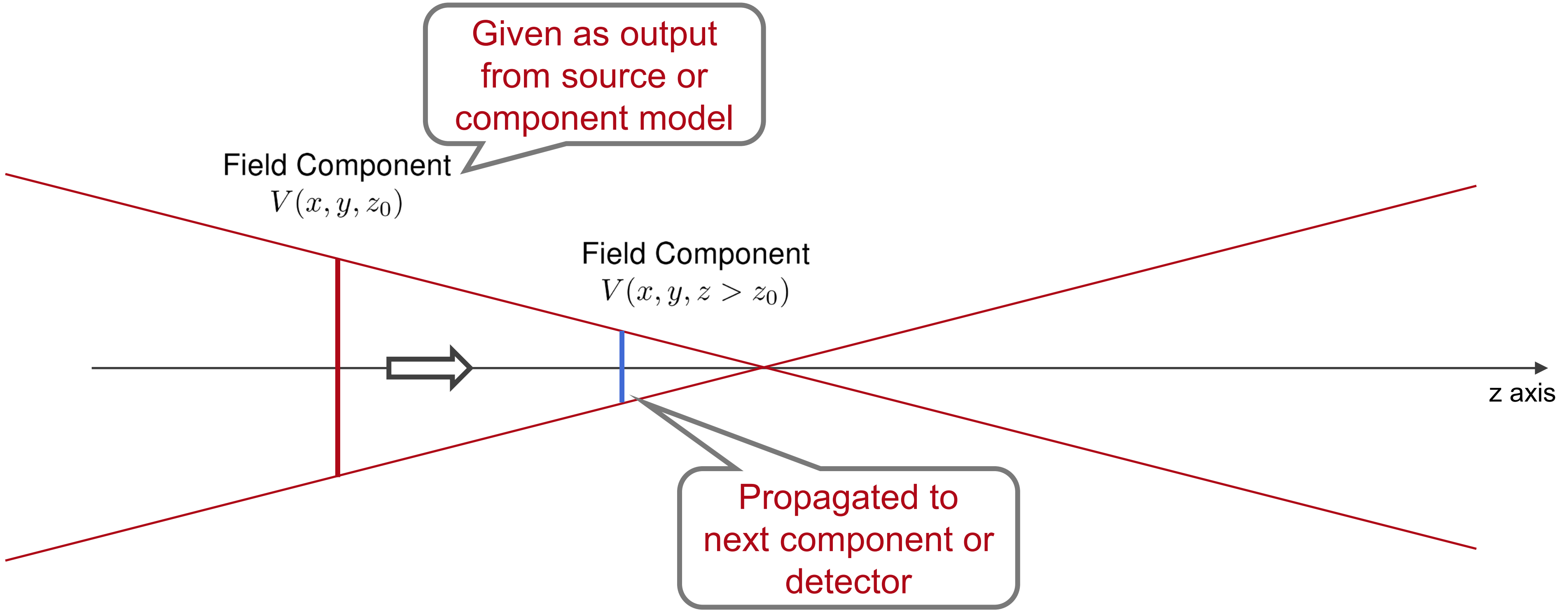
Electromagnetic Fields in Homogeneous, Isotropic Media



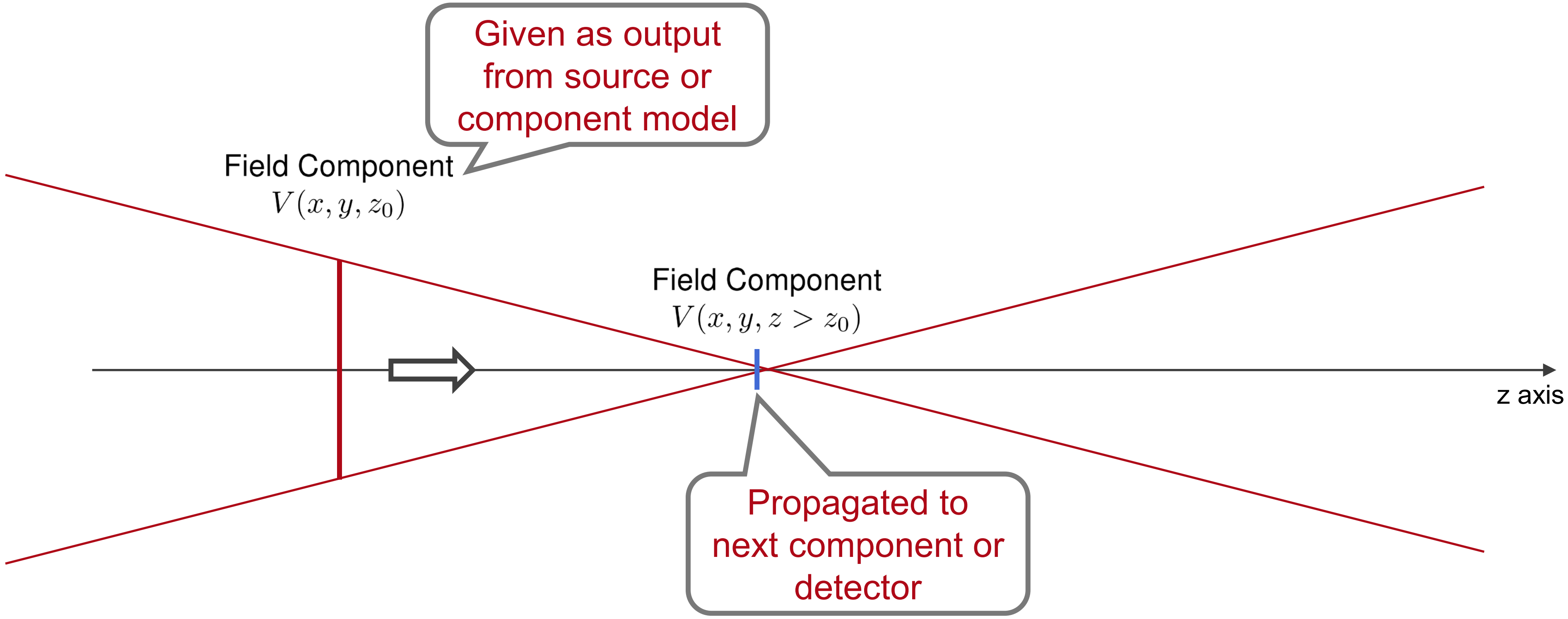
Electromagnetic Fields in Homogeneous, Isotropic Media



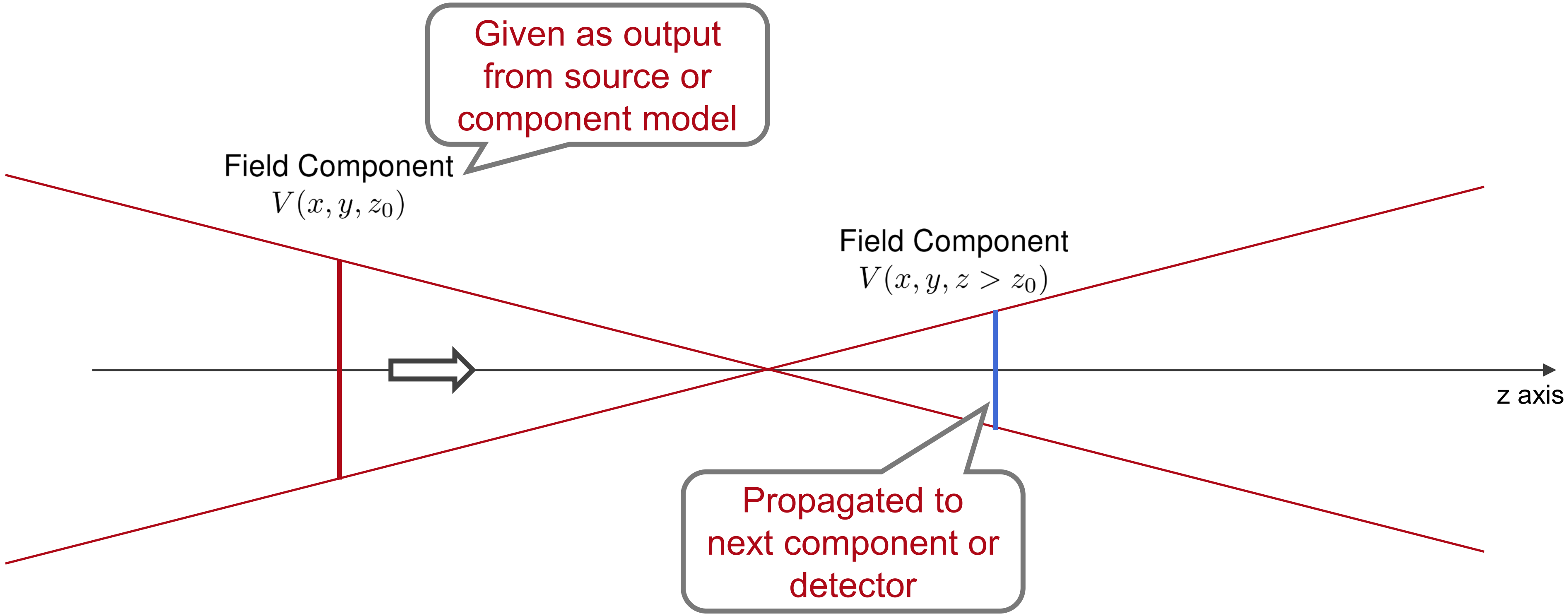
Modeling Free-Space Propagation



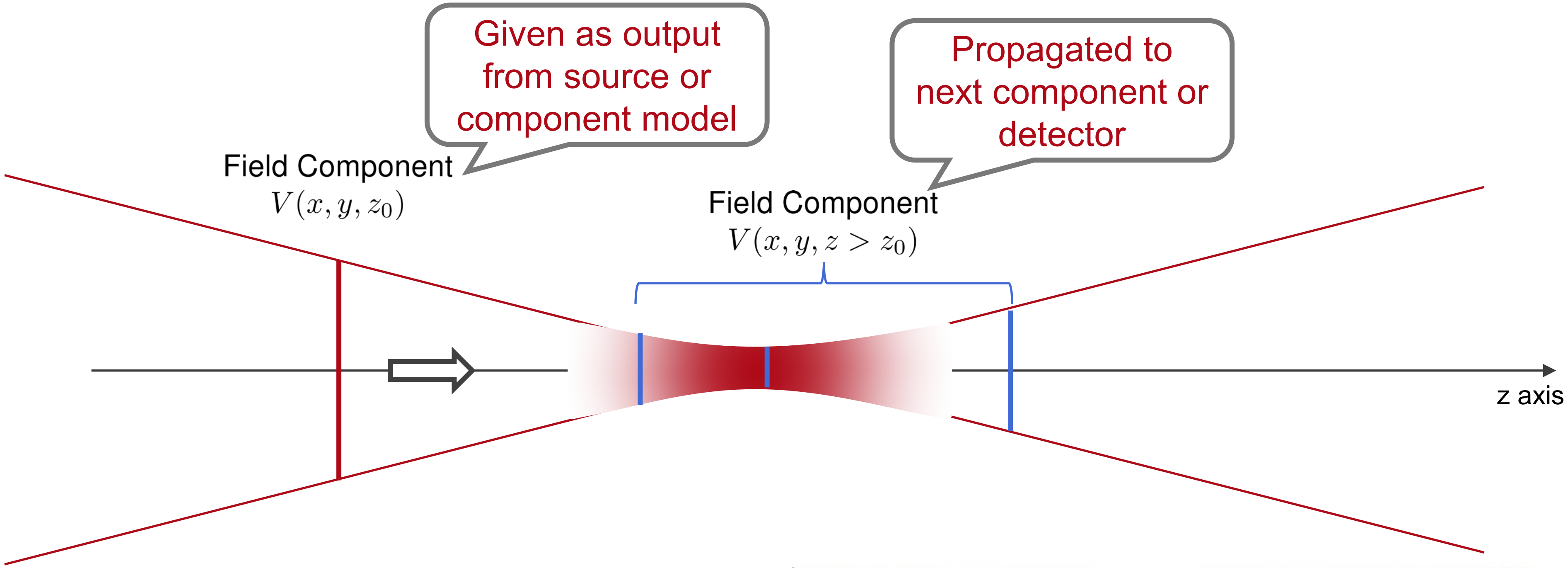
Modeling Free-Space Propagation

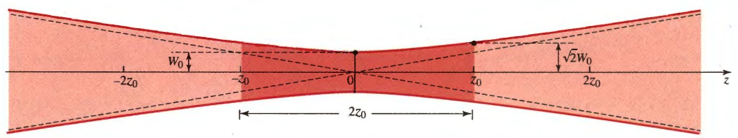


Modeling Free-Space Propagation



Modeling Free-Space Propagation





Well-known illustration for Gaussian beam propagation.

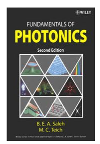
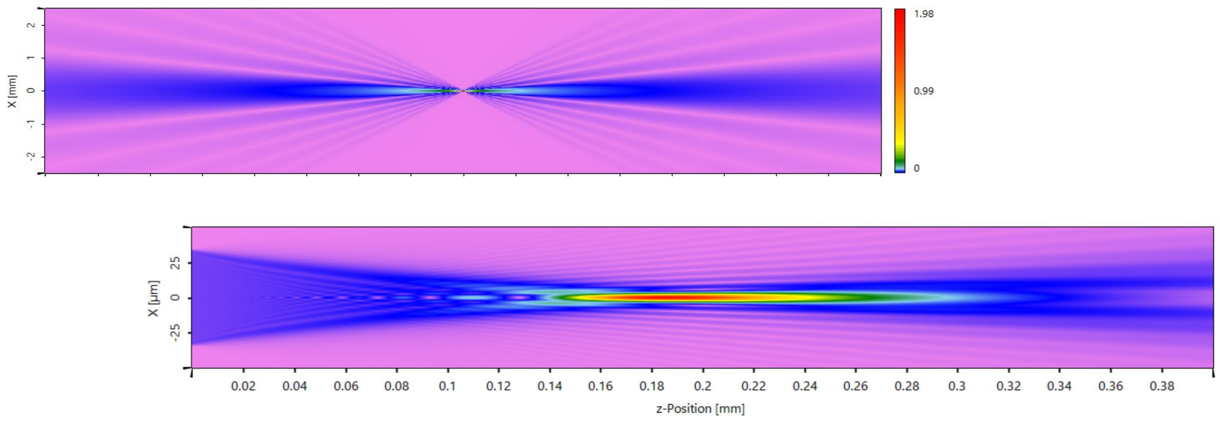
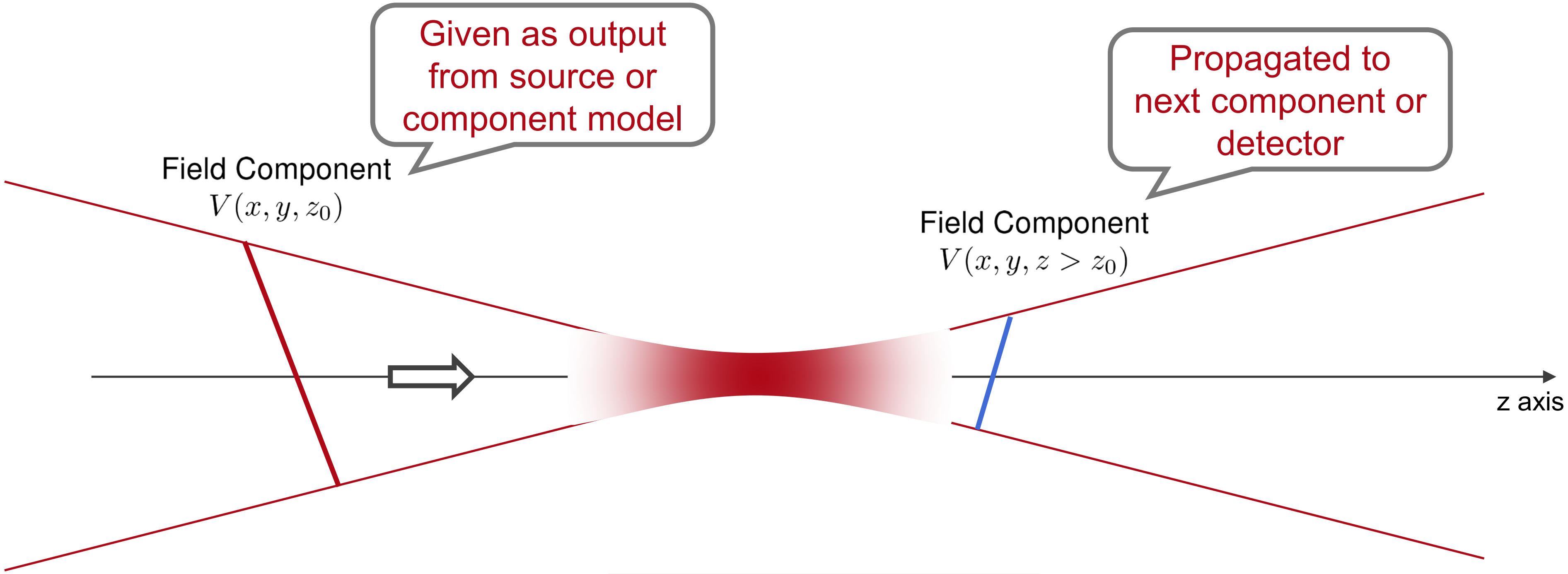


Fig. 3.1-4

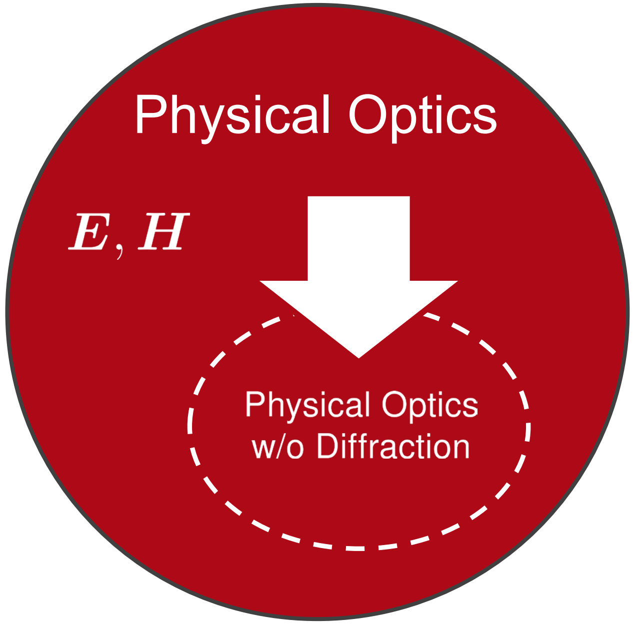


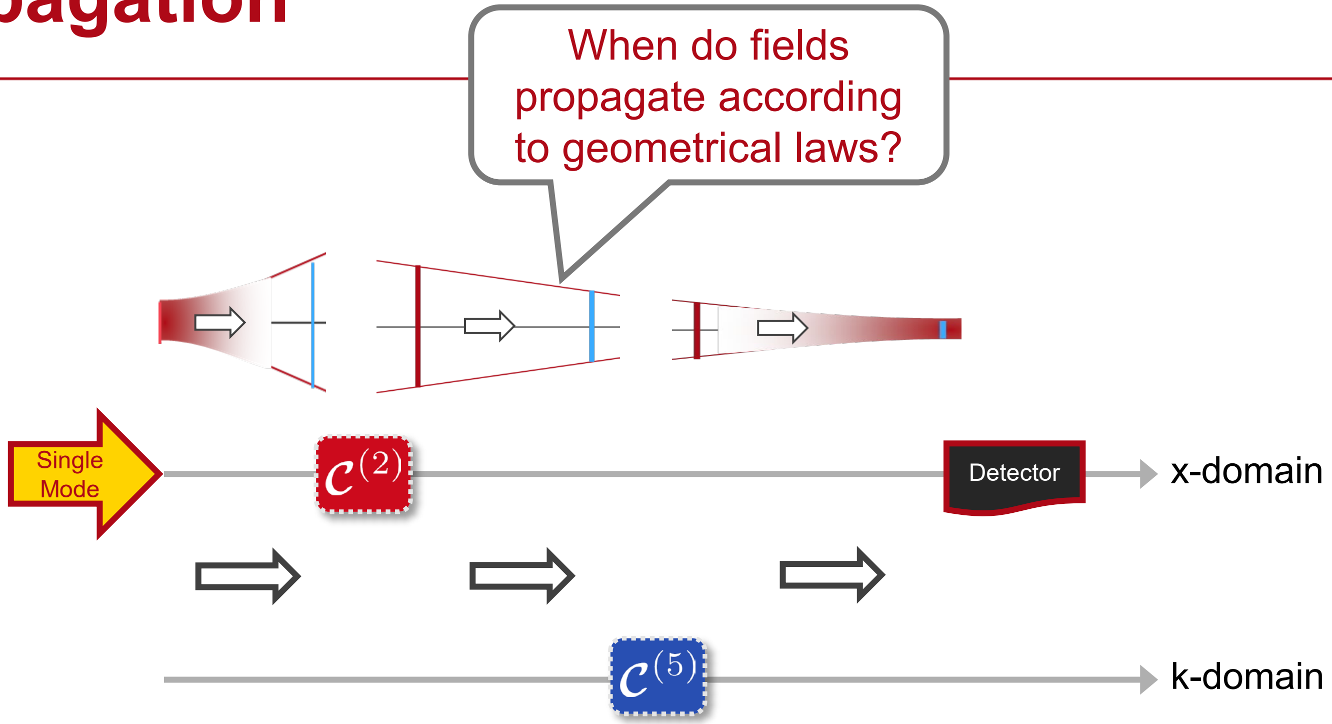
Modeling Free-Space Propagation



Theory remains valid for propagation between tilted planes.

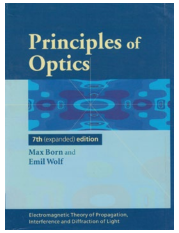
Modeling Free-Space Propagation



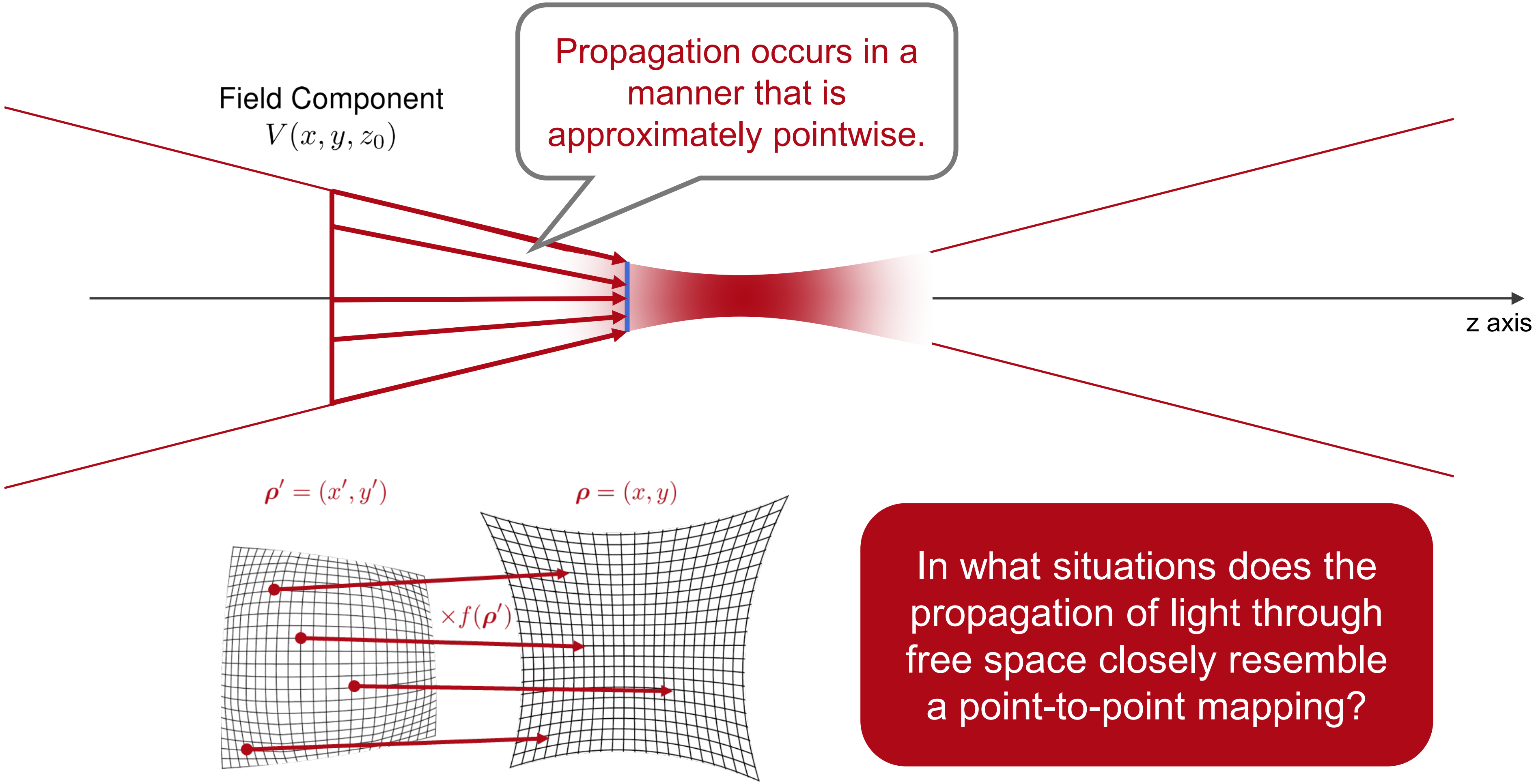


We need to identify that part of physical optics, which deals with the "***geometrical laws relating to the propagation of the 'amplitude vectors' E and H.***"

Citation
Page 125



Geometric Free-Space Propagation: Pointwise Mapping



In what situations does the propagation of light through free space closely resemble a point-to-point mapping?

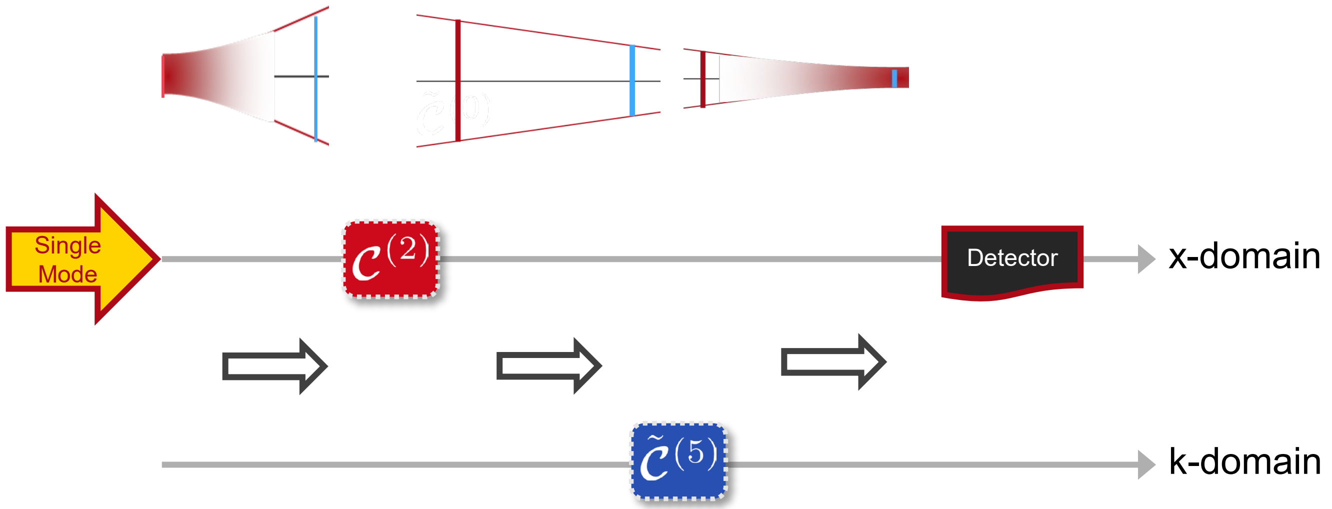
Modeling Free-Space Propagation

- The solution of Maxwell's equations in homogeneous and isotropic media provides us with a rigorous propagation operator in the k-domain:

$$\tilde{V}(k_x, k_y, z) = \left(\tilde{\mathcal{P}}\tilde{V}(z_0)\right)(k_x, k_y, z)$$
$$= \exp\left(\mathrm{i}\check{k}_z \Delta z\right) \times \tilde{V}(k_x, k_y, z_0)\,,$$

  with $\check{k}_z(k_x, k_y) = \sqrt{k_0^2 \check{n}^2 - k_x^2 - k_y^2}$ and $\Delta z = z - z_0 > 0$.

- The operation $\tilde{\mathcal{P}}\tilde{V}$ is pointwise.



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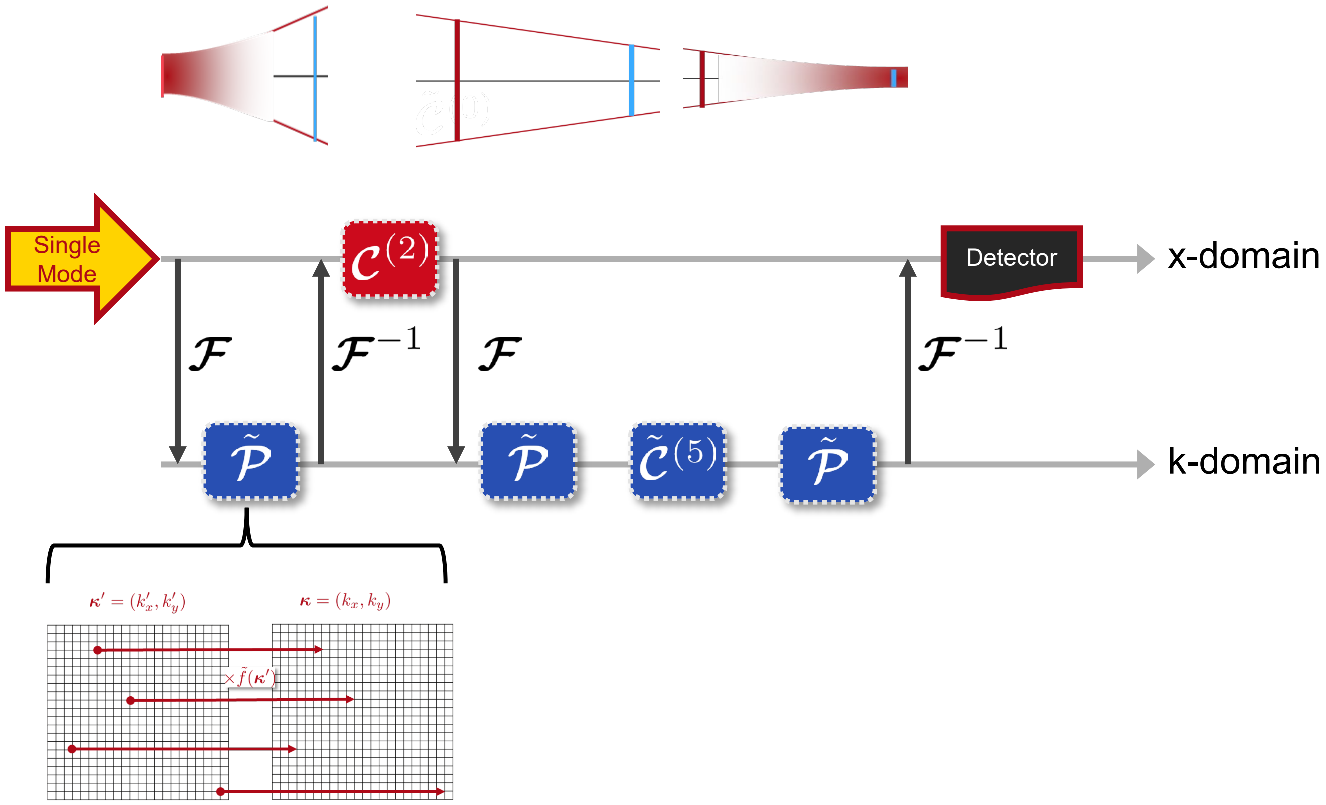

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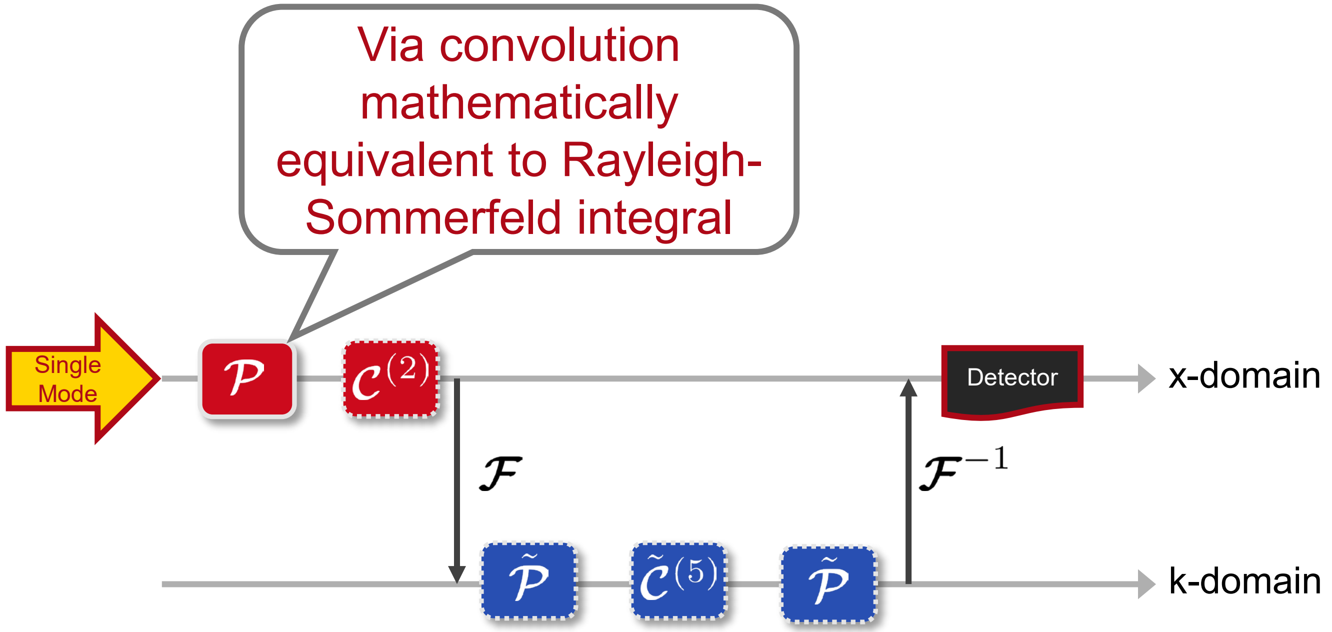

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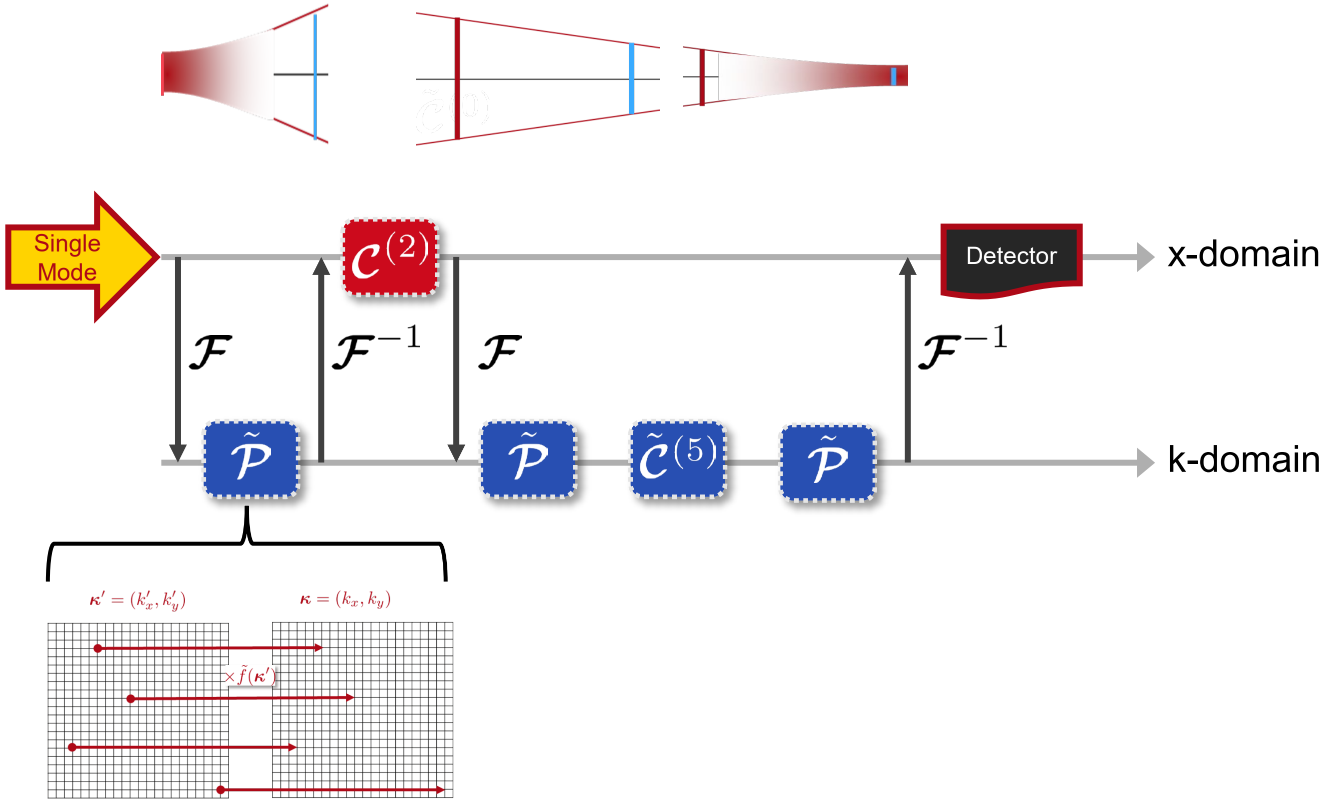

Modeling Free-Space Propagation

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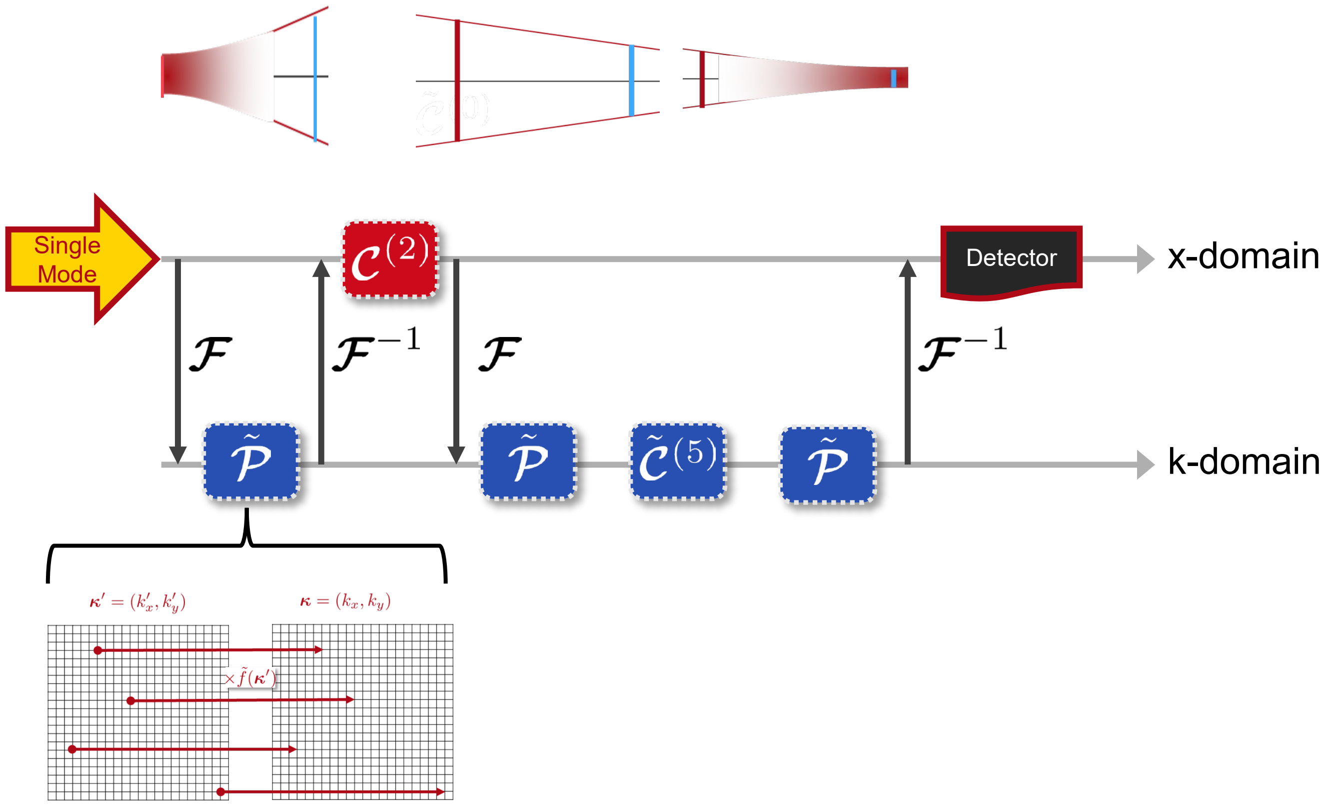
- The operation $\tilde{\mathcal{P}}\tilde{V}$ is pointwise.
- Hence, the propagation in the x-domain can only adhere to geometrical laws when the forward and inverse Fourier transforms exhibit nearly pointwise behavior.



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$$\begin{aligned}\tilde{V}(k_x,k_y,z) &= \big(\tilde{\mathcal{P}}\tilde{V}(z_0)\big)(k_x,k_y,z)\\ &= \exp\left(\mathrm{i}\check{k}_z\Delta z\right)\times\tilde{V}(k_x,k_y,z_0)\,,\end{aligned}$$

with $\check{k}_z(k_x,k_y) = \sqrt{k_0^2\check{n}^2 - k_x^2 - k_y^2}$ and $\Delta z = z - z_0 > 0$.

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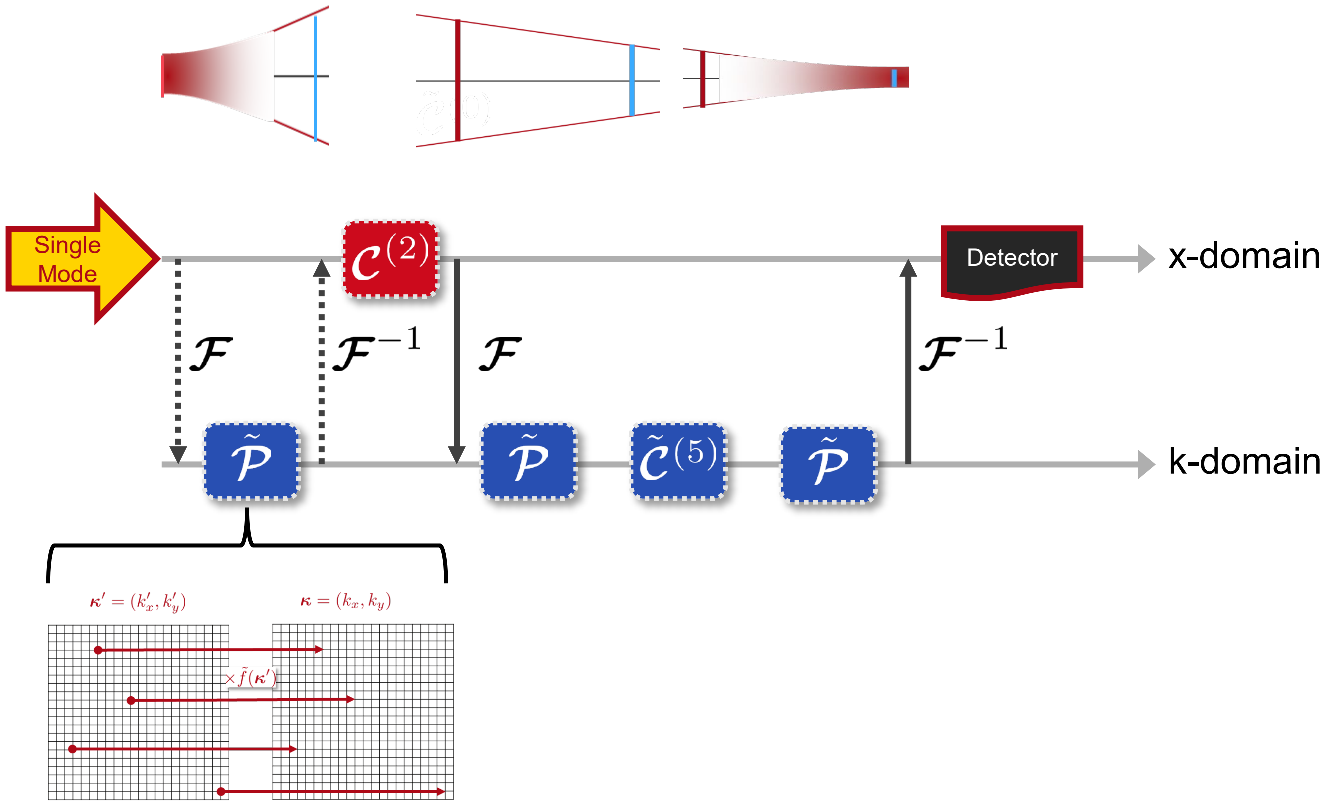
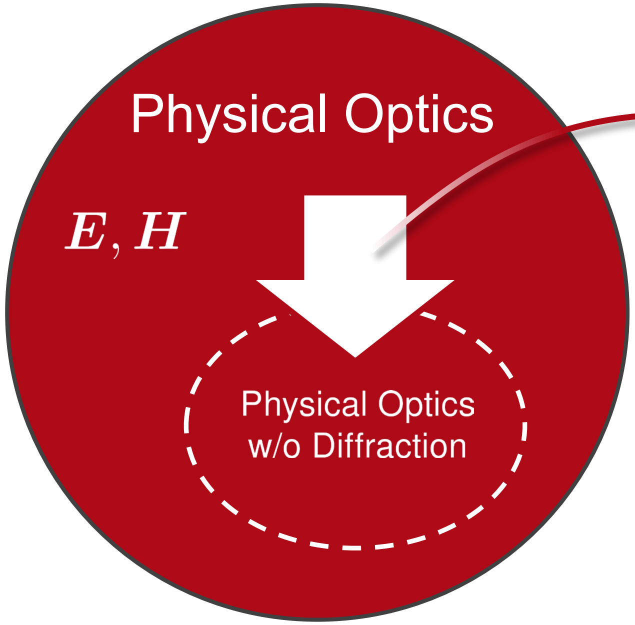


Geometrical Optics for Electromagnetic Fields



When does free-space propagation approximately reduce to a pointwise mapping?

When does the Fourier transform behave in a pointwise manner?

We need to identify that part of physical optics, which deals with the "***geometrical laws relating to the propagation of the 'amplitude vectors' E and H.***"

Citation
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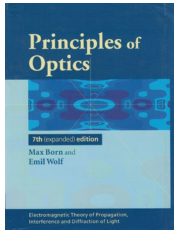
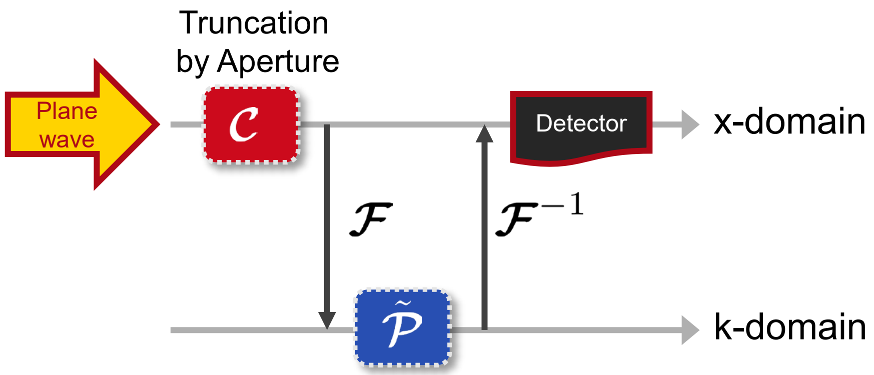


Example: Diffraction at Circular Aperture

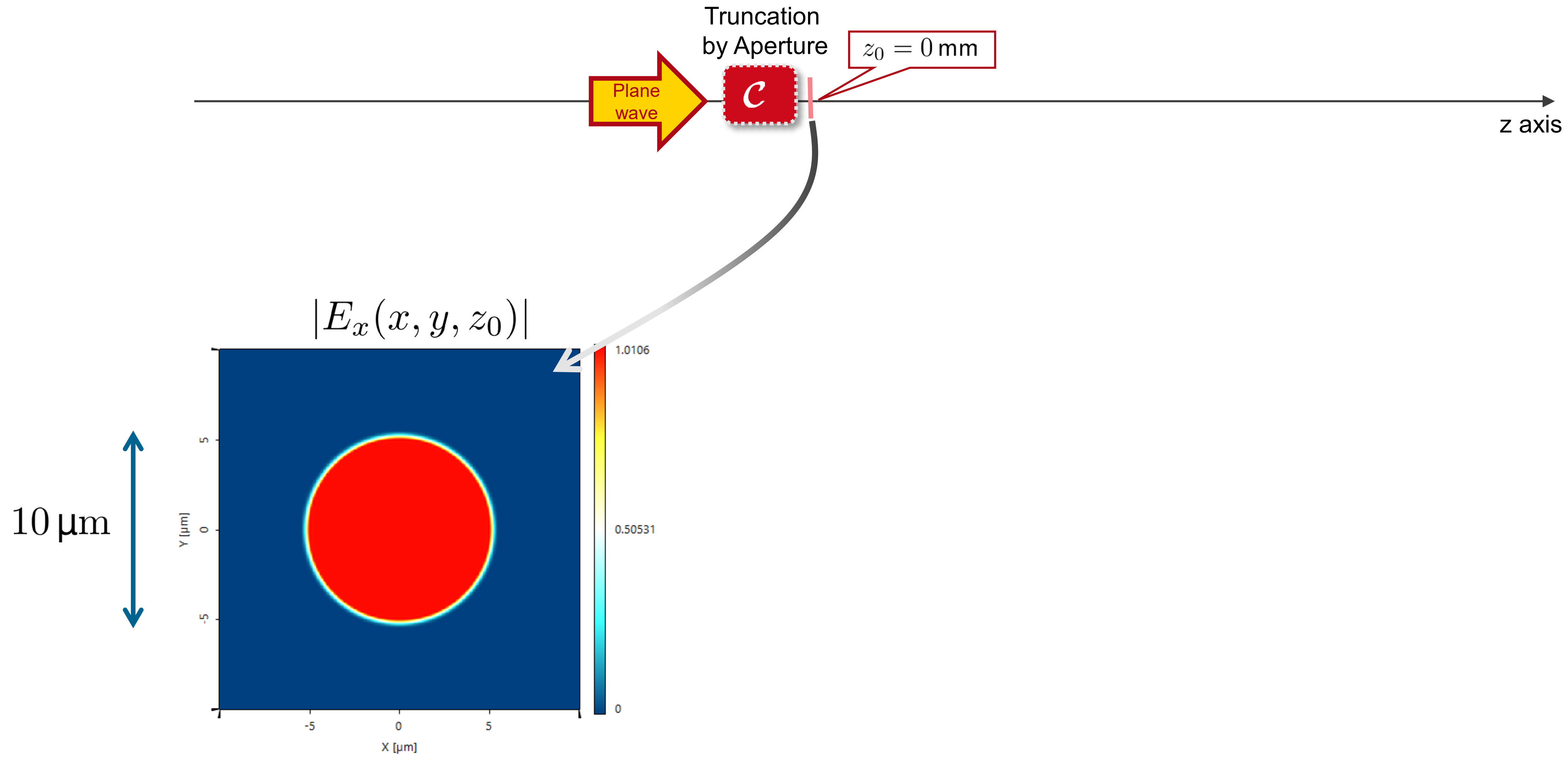


When does free-space propagation approximately reduce to a pointwise mapping?

⇓

When does the Fourier transform behave in a pointwise manner?

Example: Diffraction at Circular Aperture



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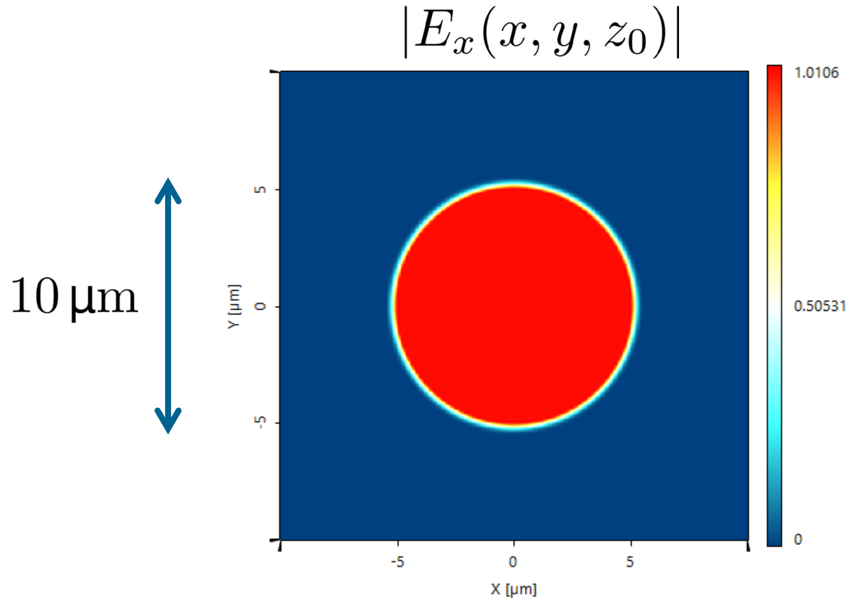

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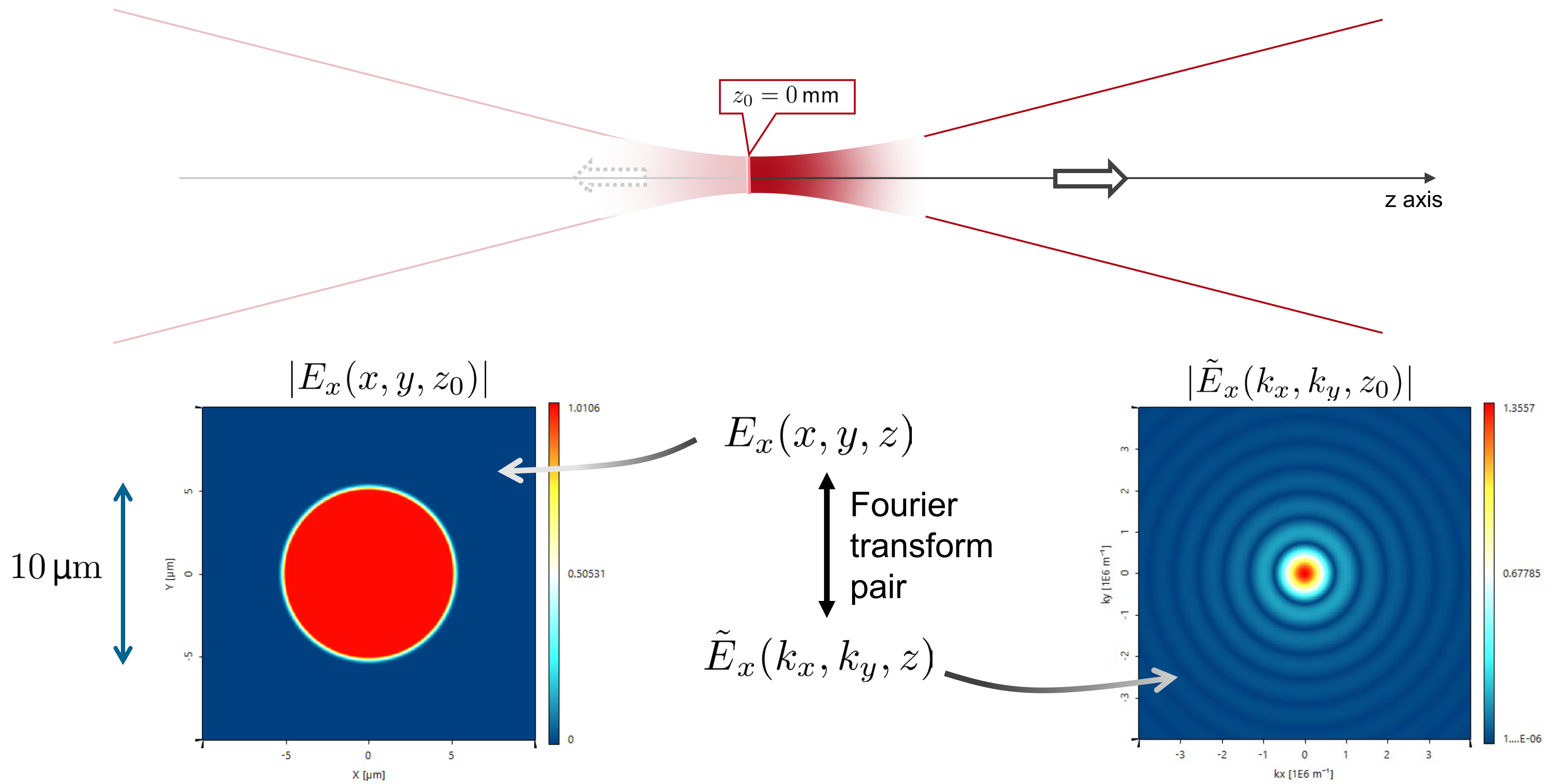
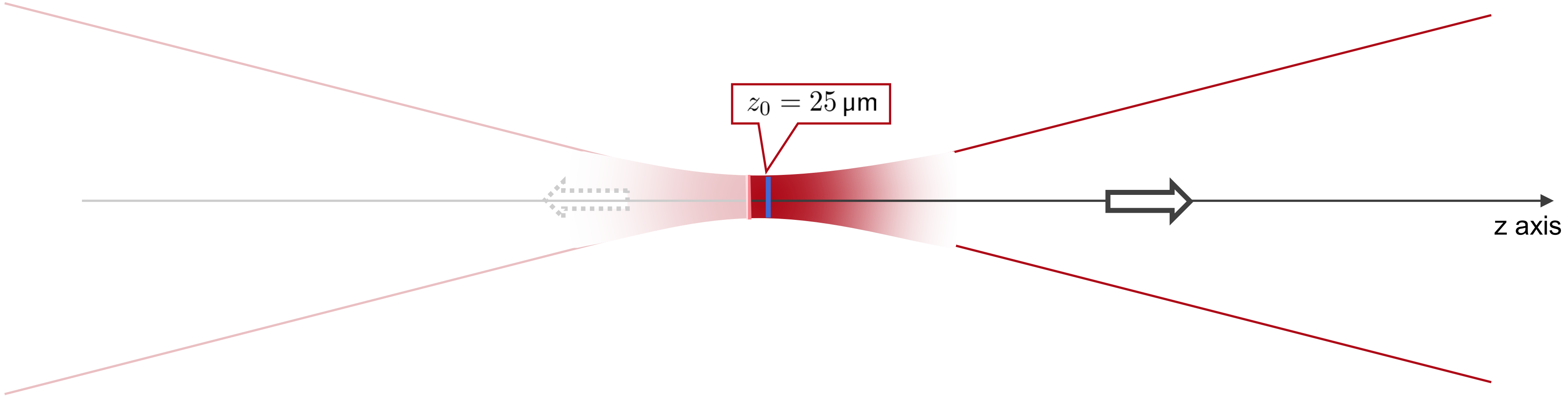
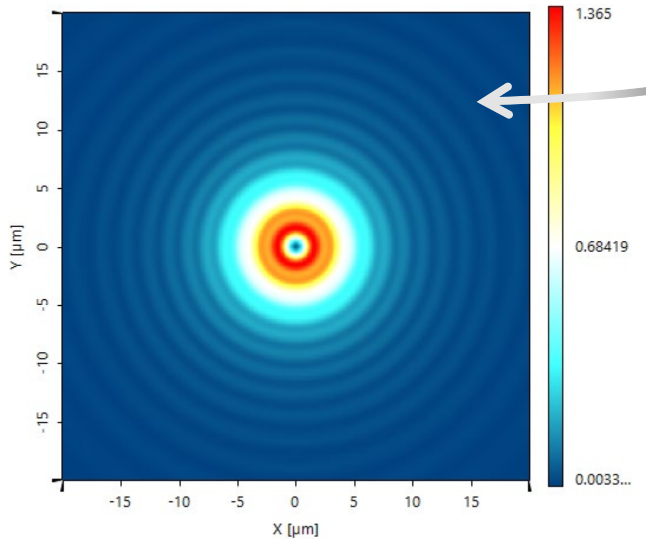


Example: Diffraction at Circular Aperture



$|E_x(x,y,z)|$

40 µm

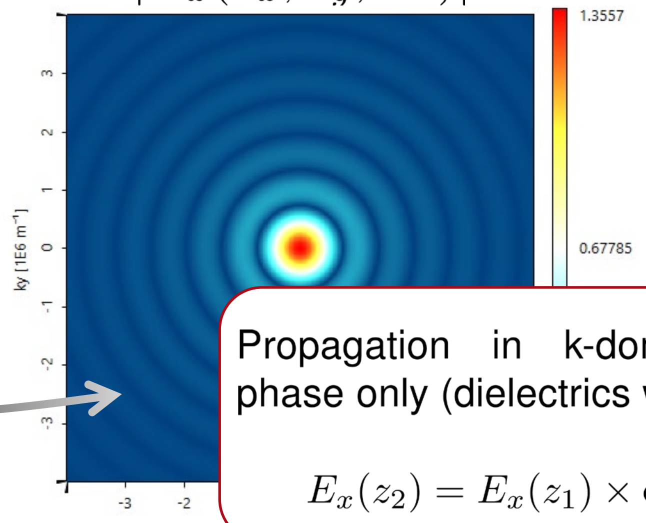


$E_x(x,y,z)$

Fourier transform pair

$\tilde{E}_x(k_x,k_y,z)$

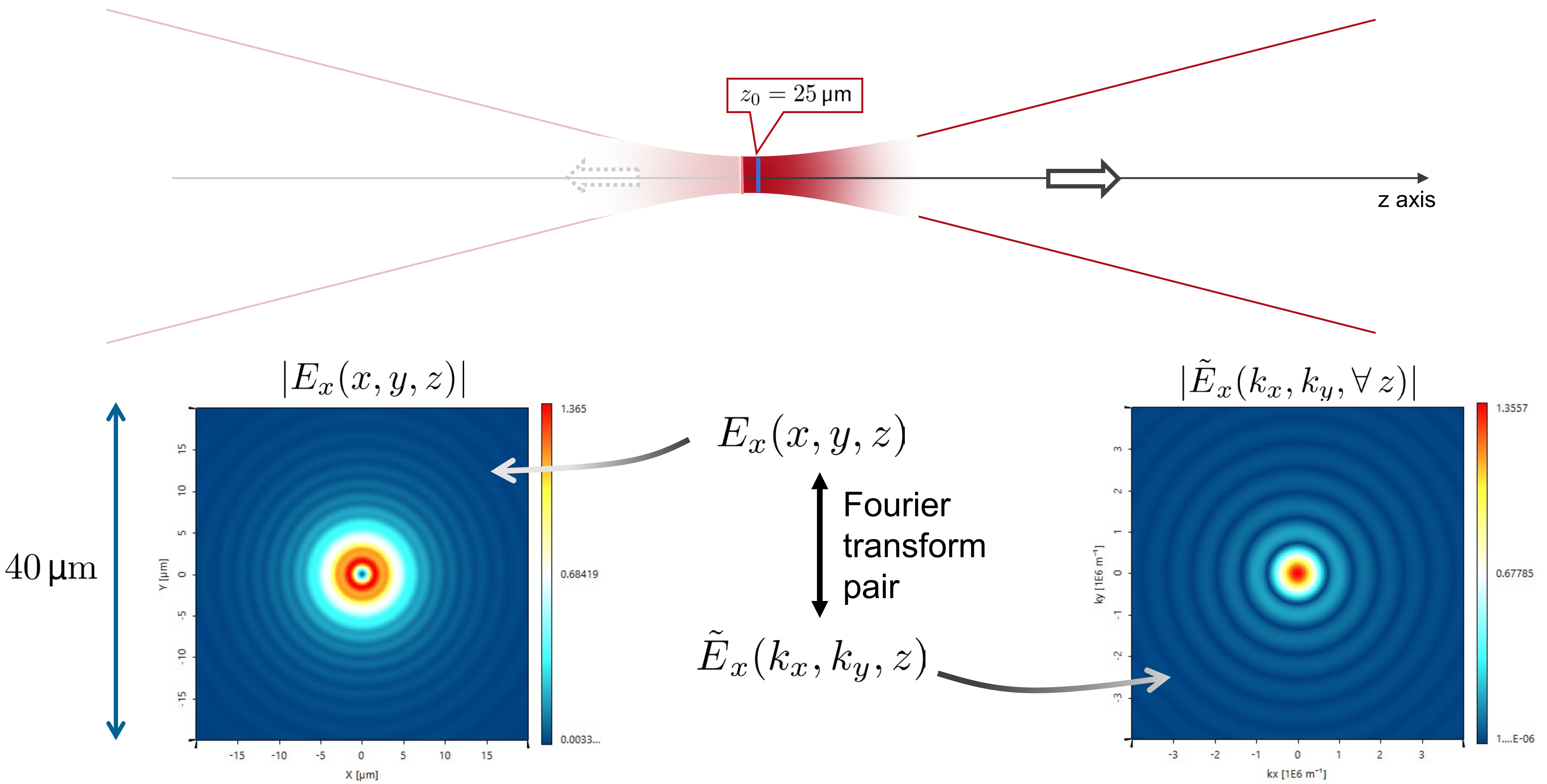
$|\tilde{E}_x(k_x,k_y,\forall\, z)|$



Propagation in k-domain changes phase only (dielectrics w/o damping):

$$E_x(z_2) = E_x(z_1) \times \exp\left[\mathrm{i}k_z \Delta z\right]$$

Example: Diffraction at Circular Aperture



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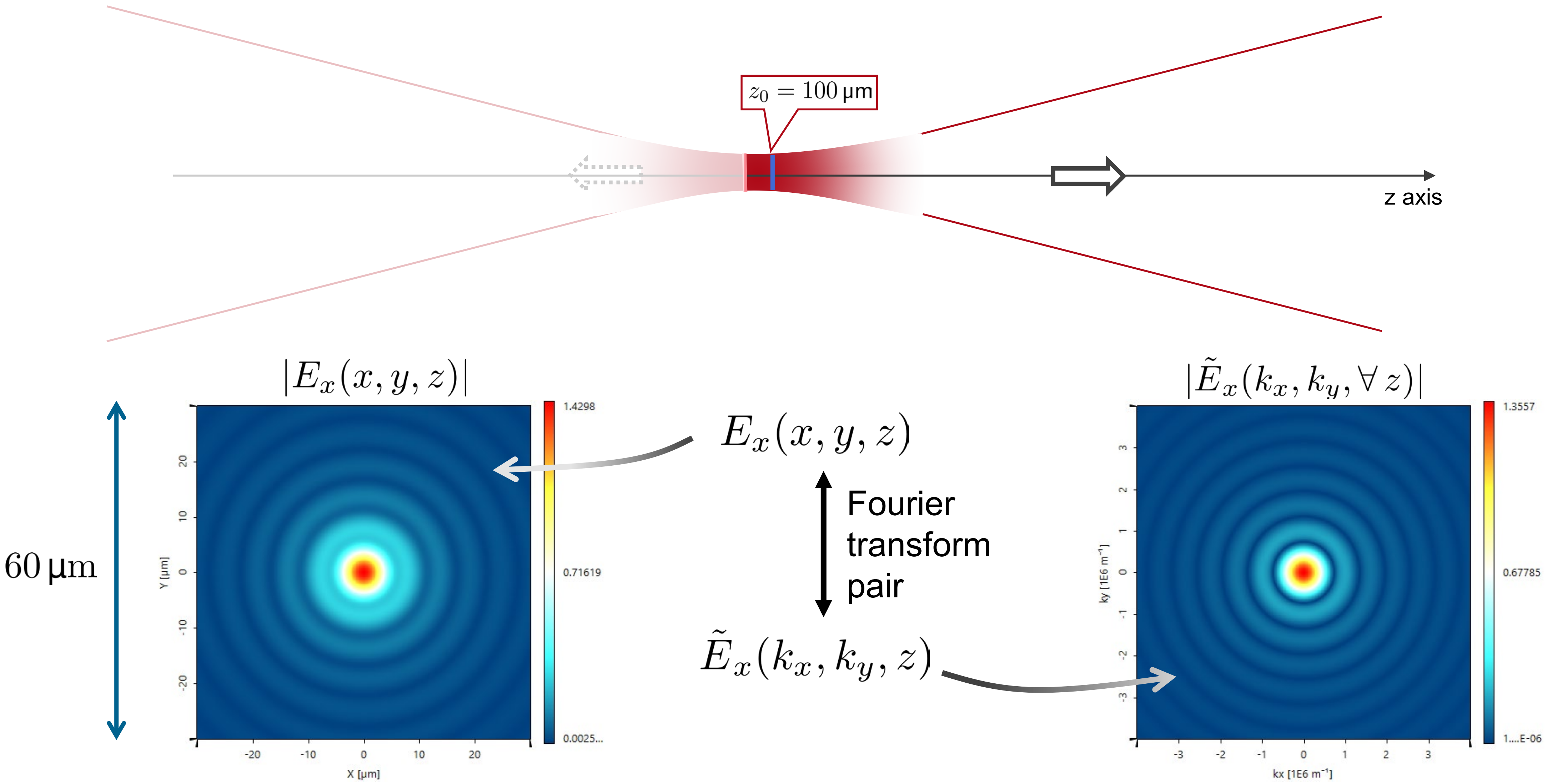

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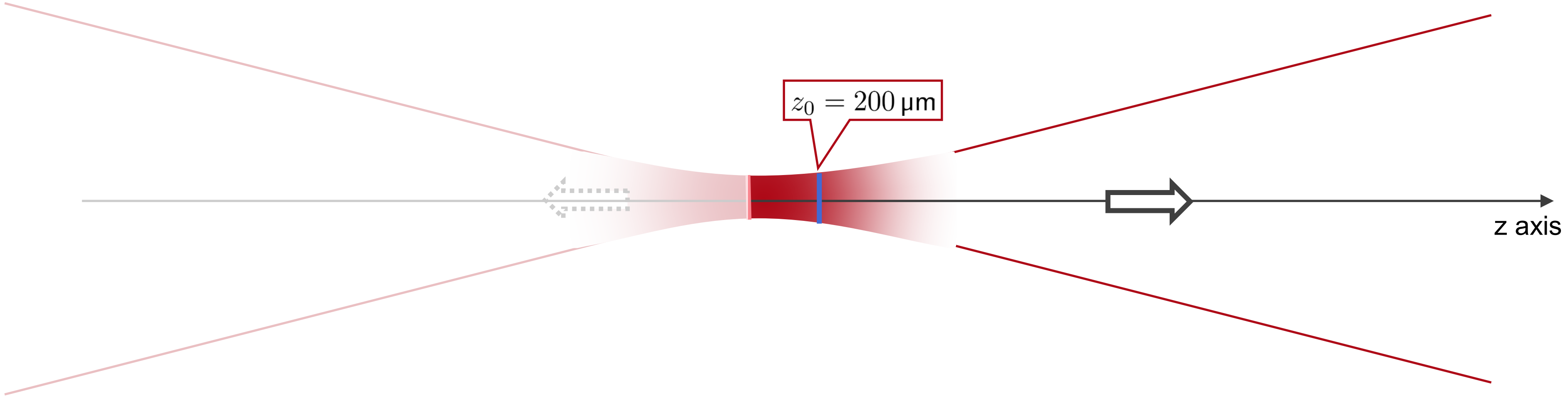

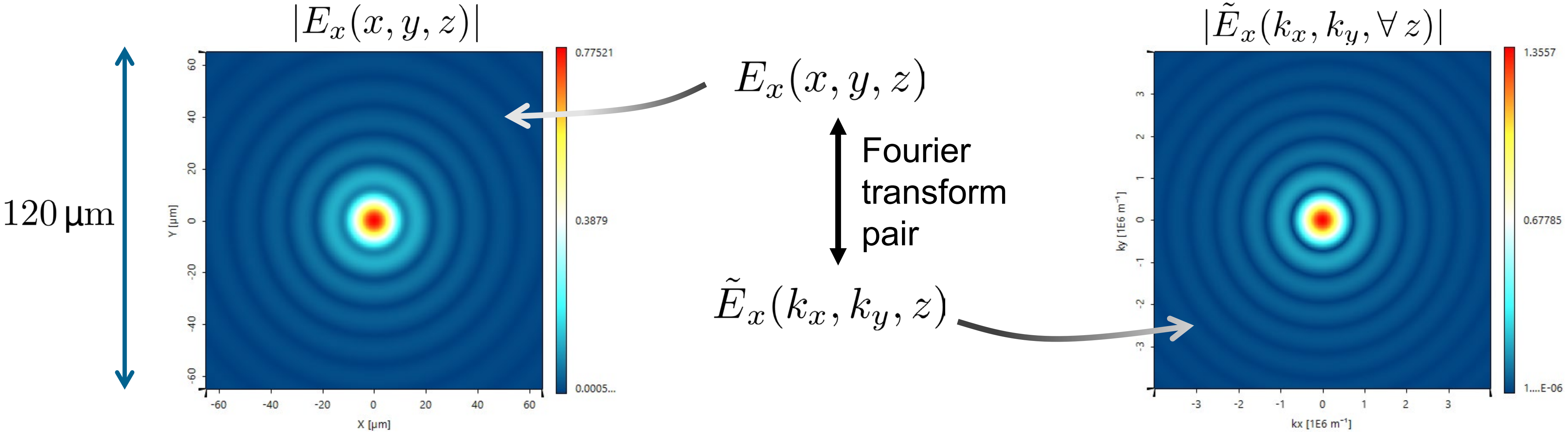

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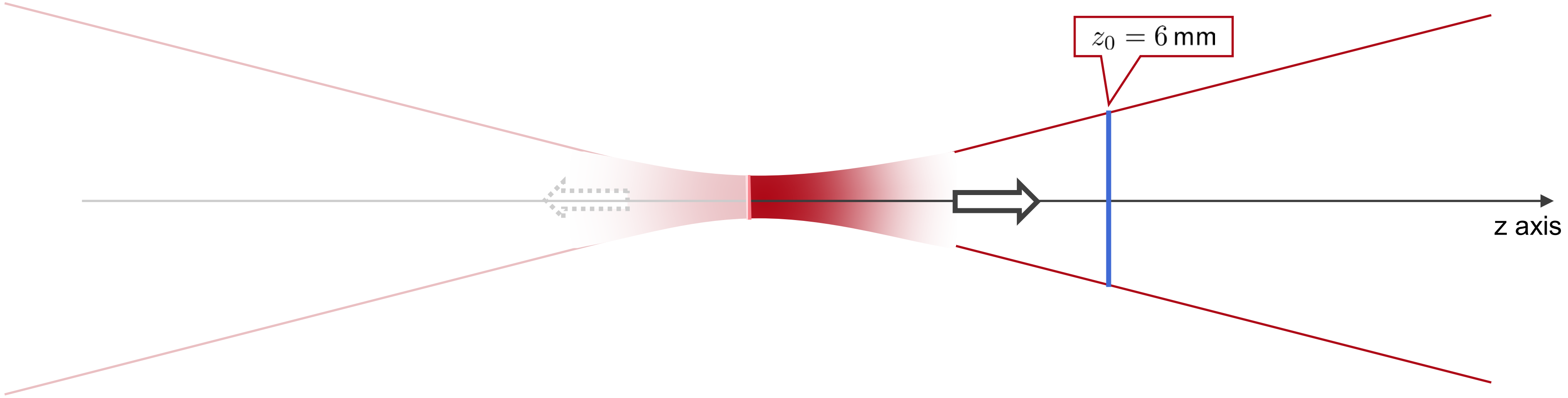

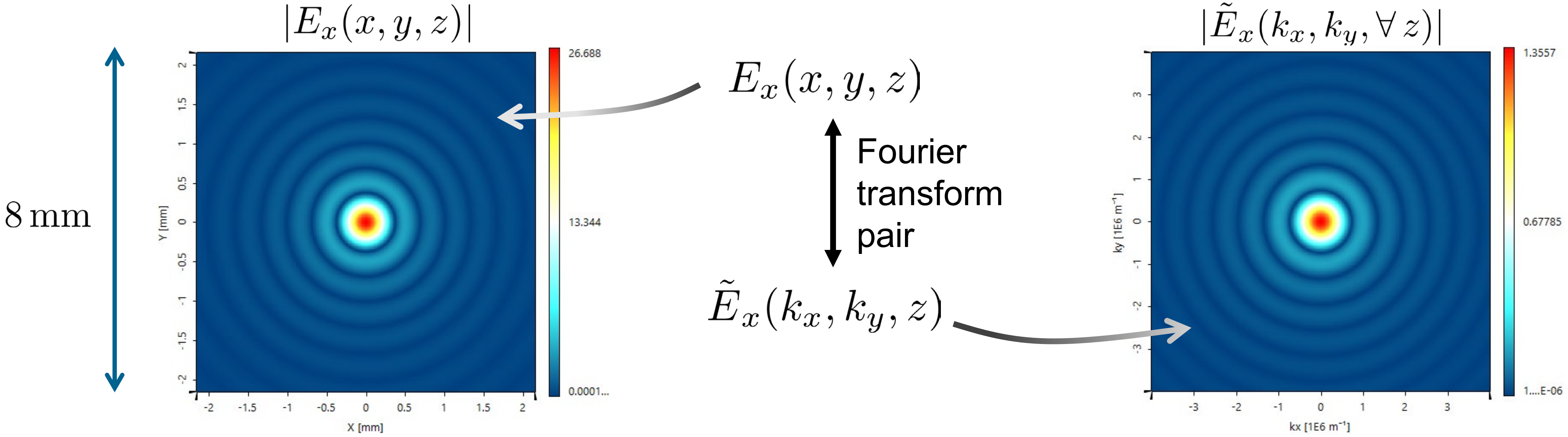

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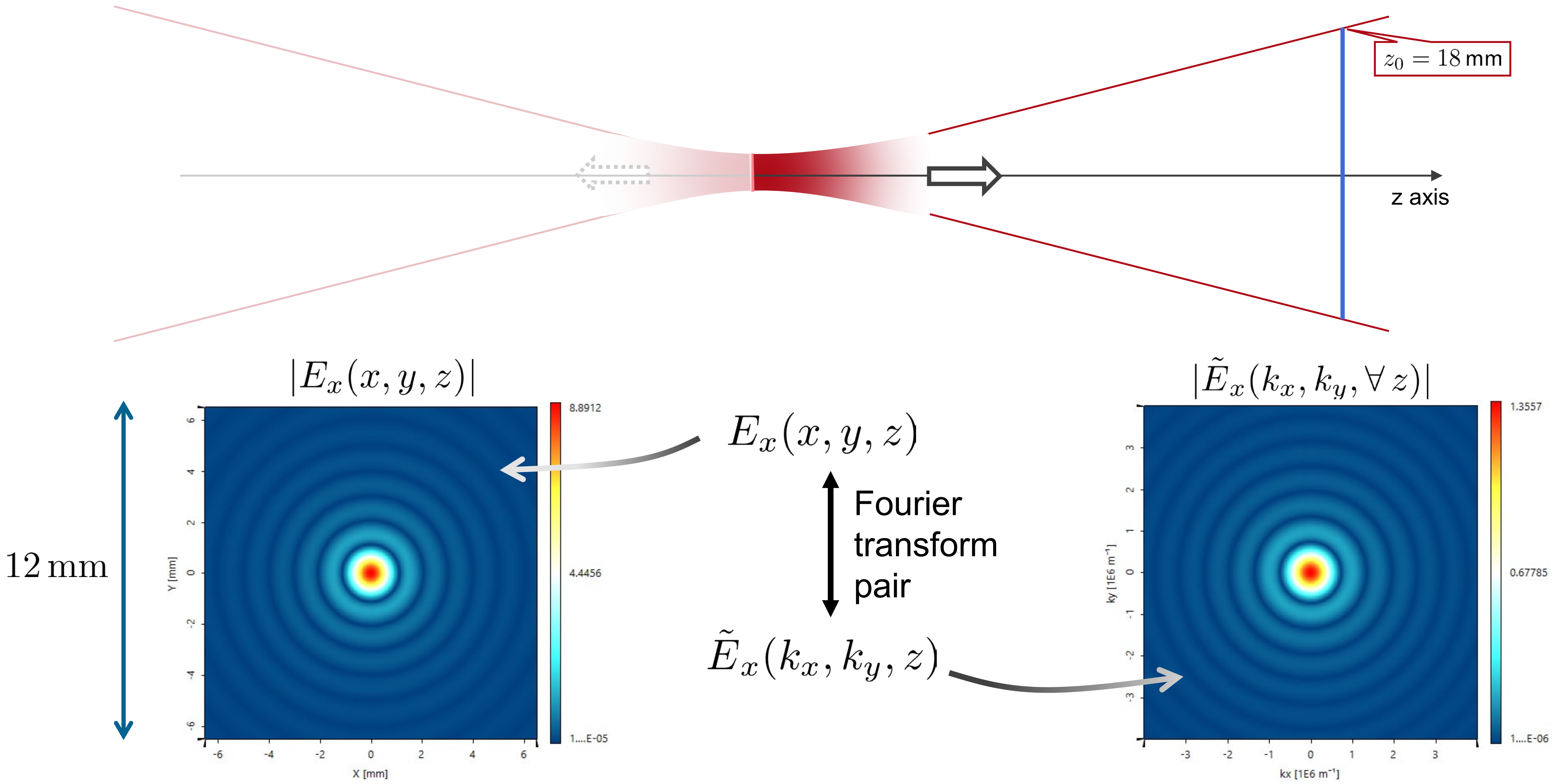
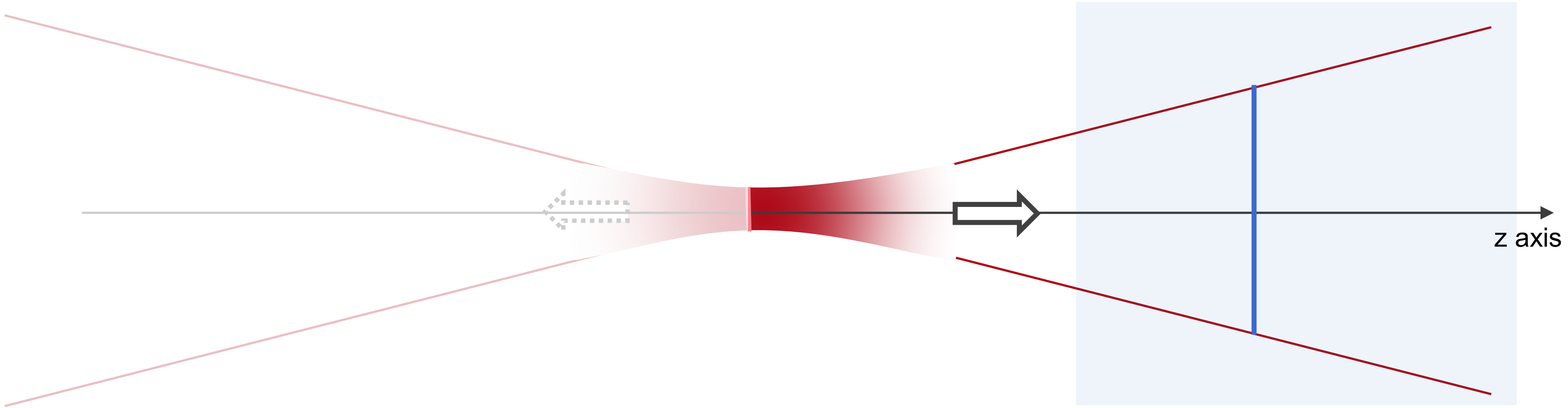
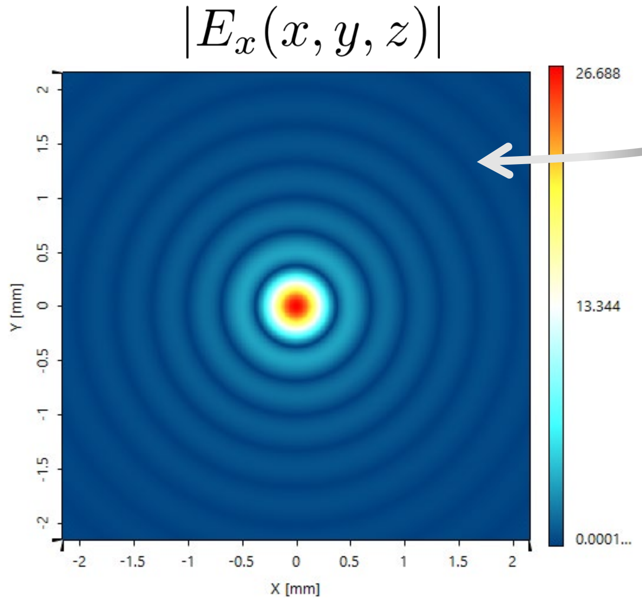


Example: Diffraction at Circular Aperture: Far Field Zone

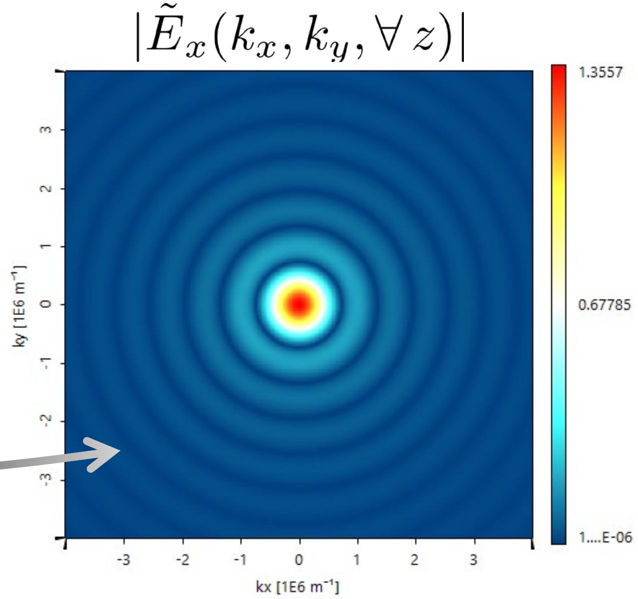




$E_x(x,y,z)$

Fourier transform pair

$\tilde{E}_x(k_x,k_y,z)$



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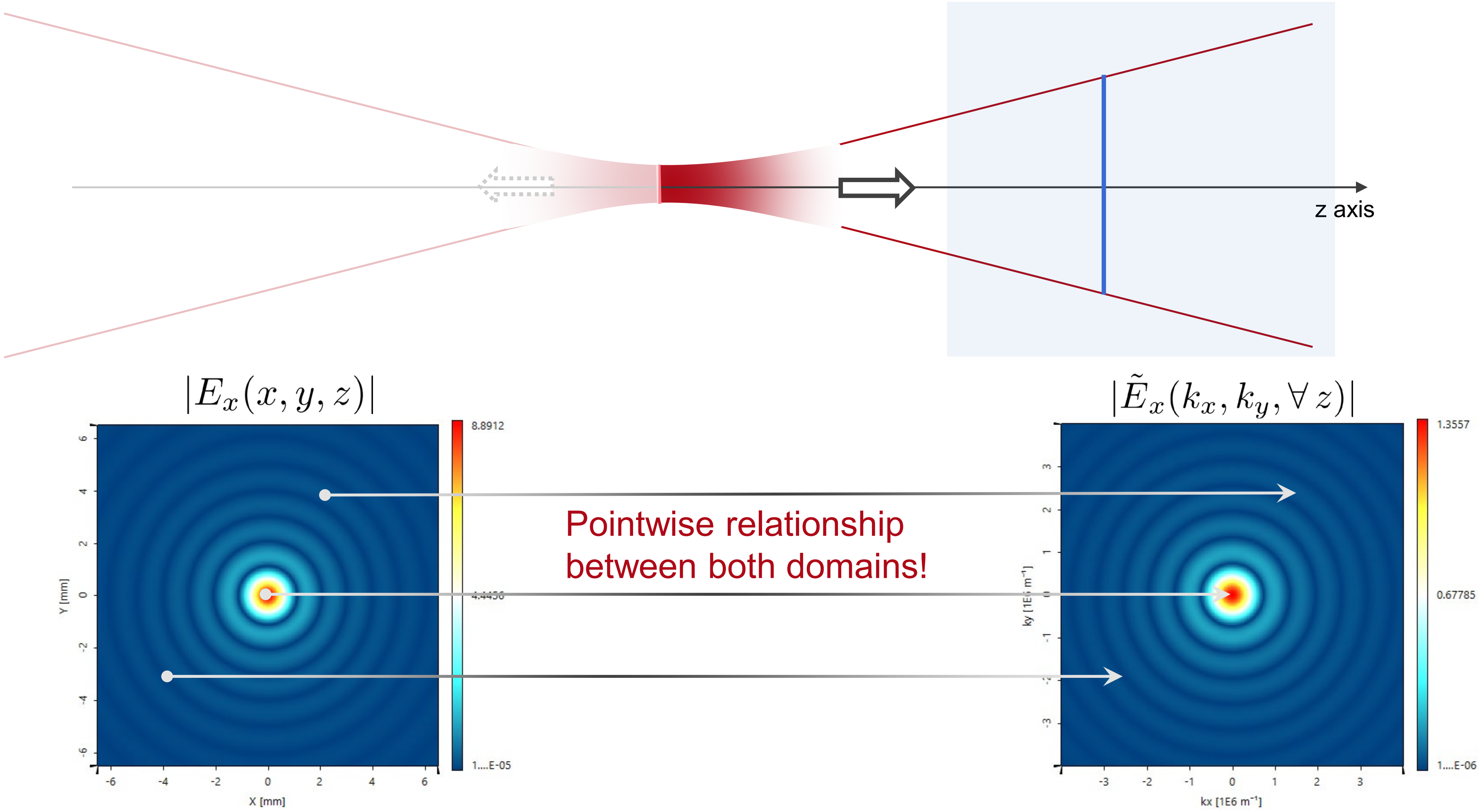
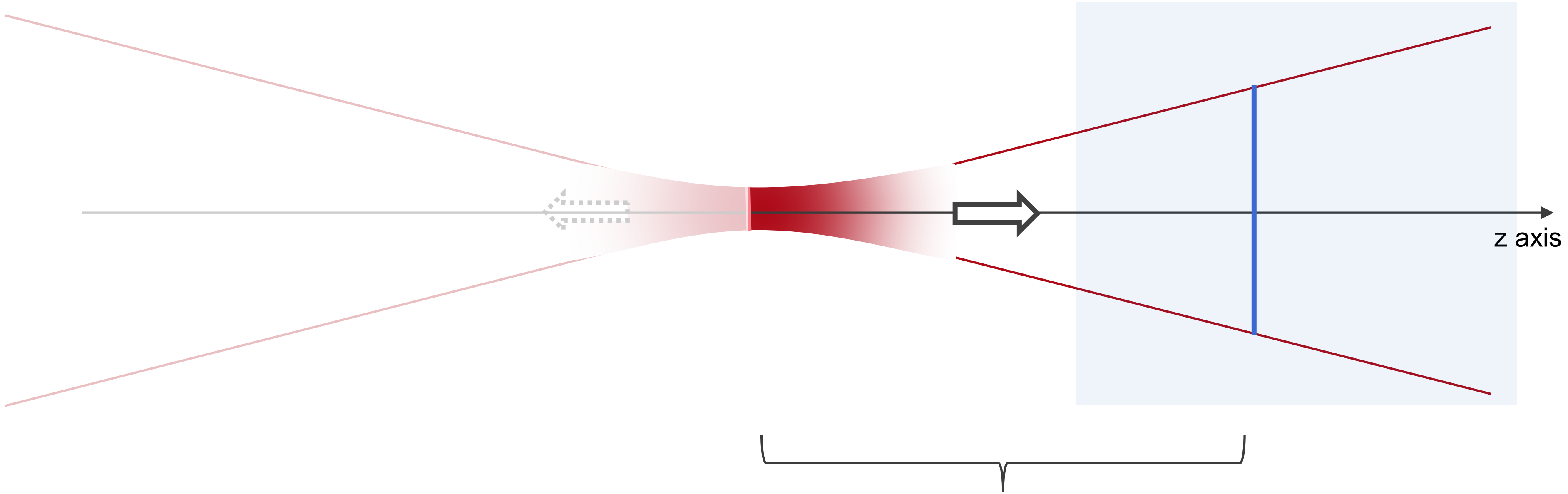
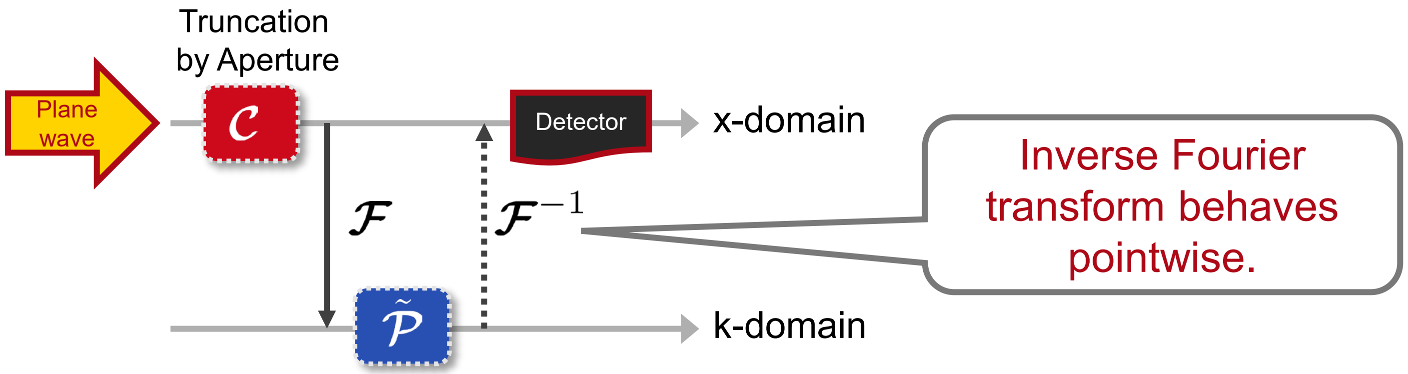
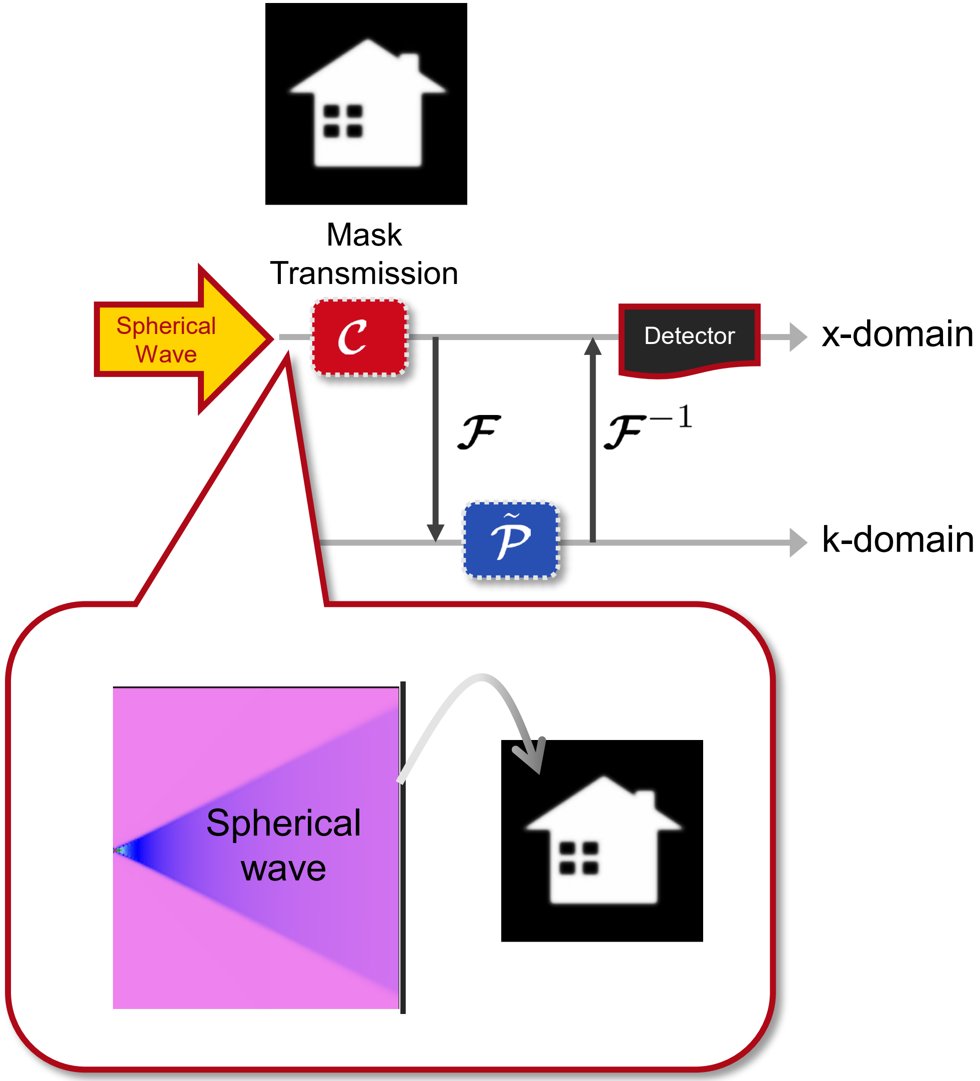


Example: Diffraction at Circular Aperture: Far Field Zone





Example: Illumination of Slide with Spherical Wave

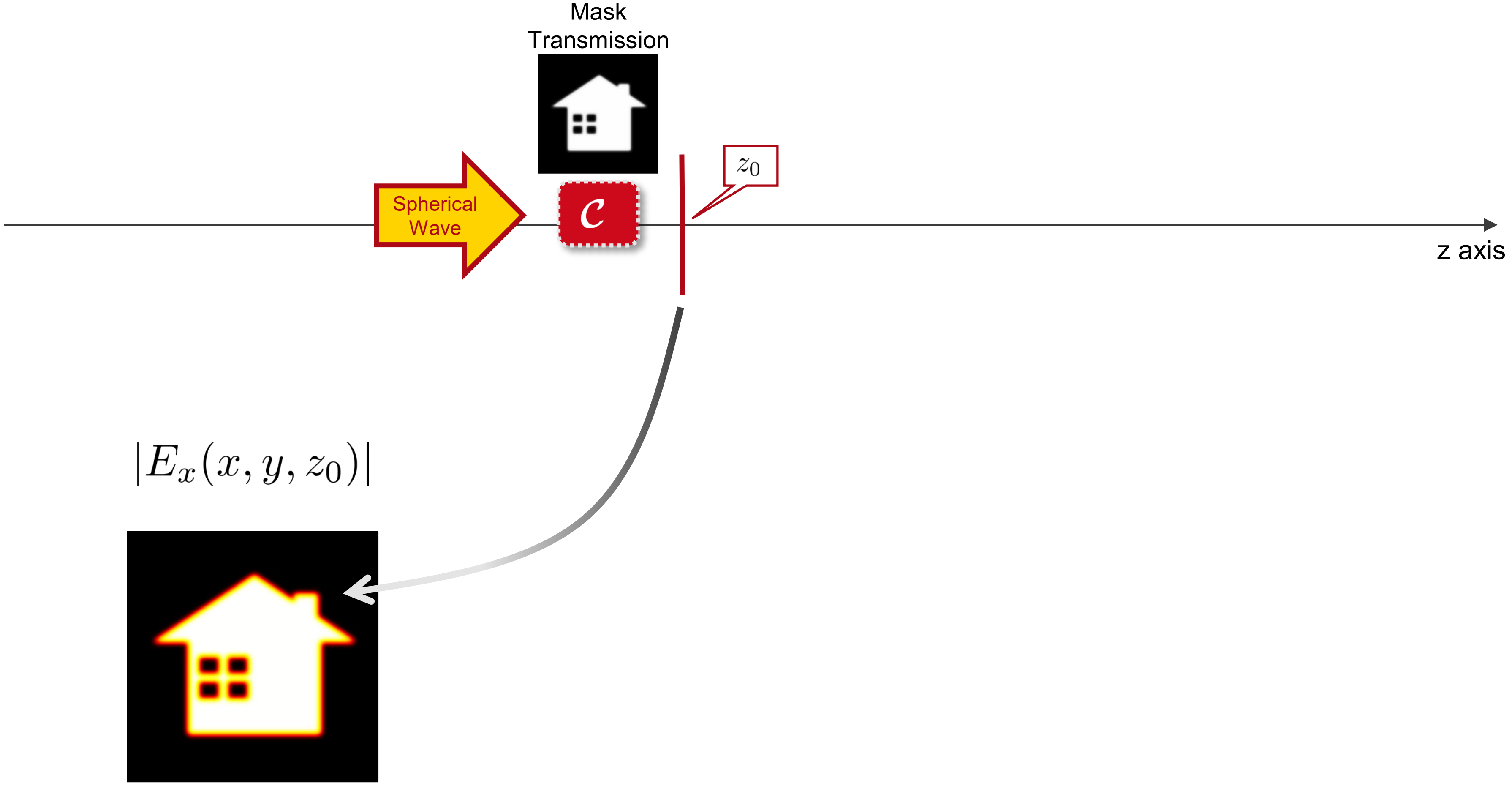


When does free-space propagation approximately reduce to a pointwise mapping?

⇓

When does the Fourier transform behave in a pointwise manner?

Example: Propagation Behind Mask



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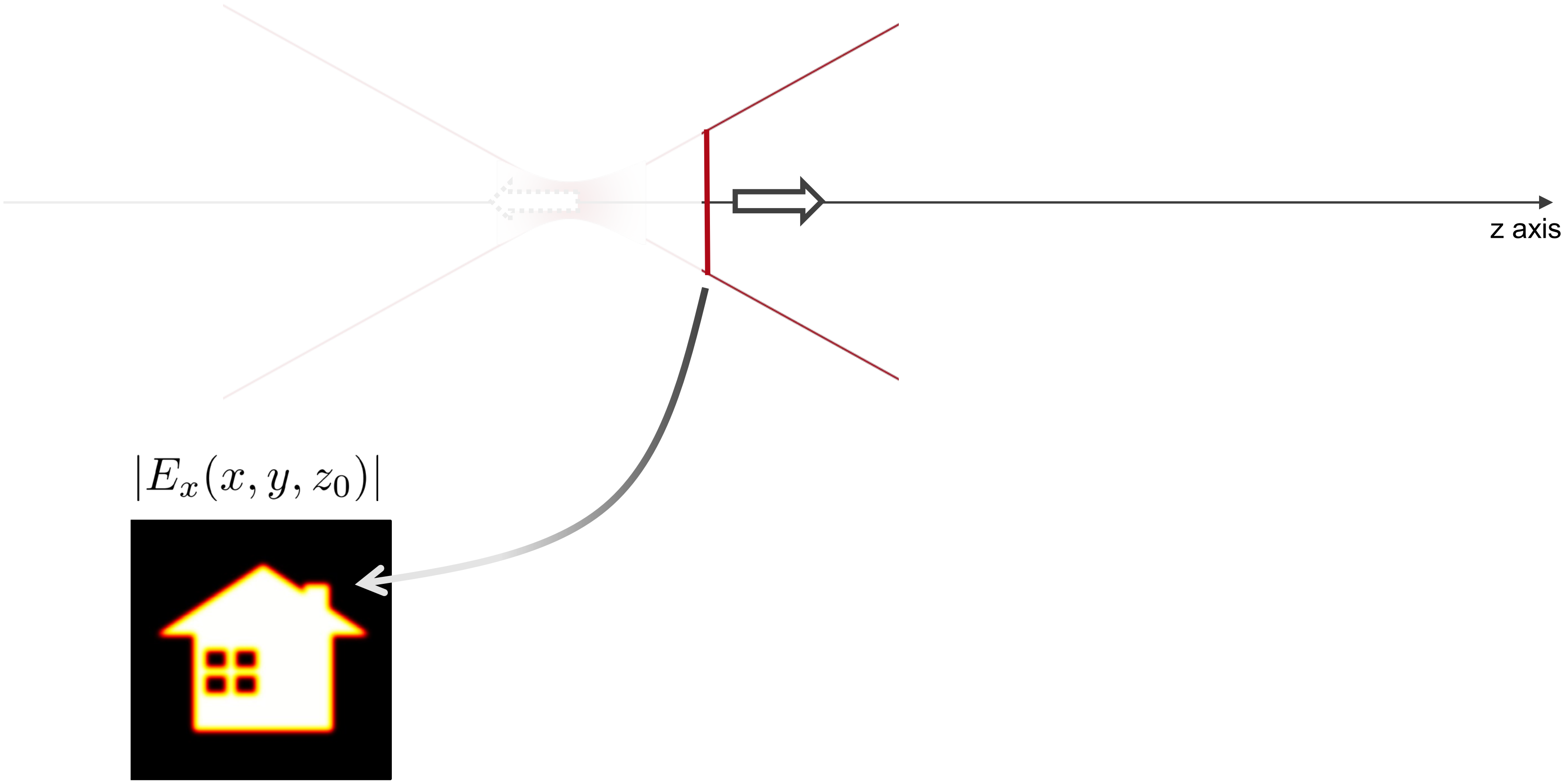

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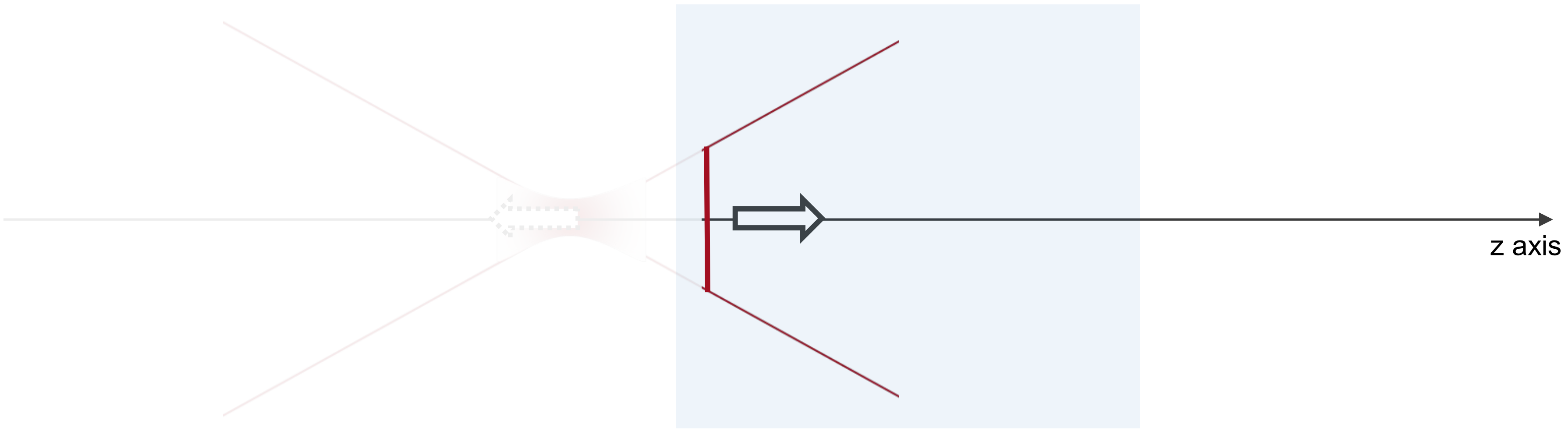


$|E_x(x, y, z_0)|$

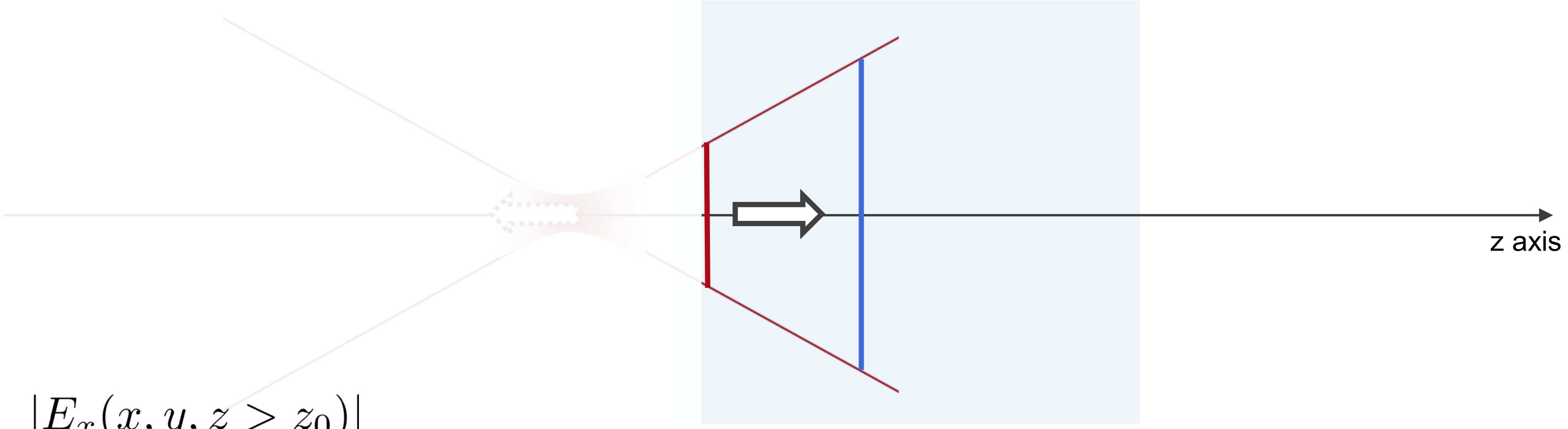
$E_x(x, y, z)$

Fourier transform pair

$\tilde{E}_x(k_x, k_y, z)$

$|\tilde{E}_x(k_x, k_y, \forall\, z)|$

Example: Propagation Behind Mask



$|E_x(x, y, z > z_0)|$

$E_x(x, y, z)$

Fourier transform pair

$\tilde{E}_x(k_x, k_y, z)$

$|\tilde{E}_x(k_x, k_y, \forall\, z)|$

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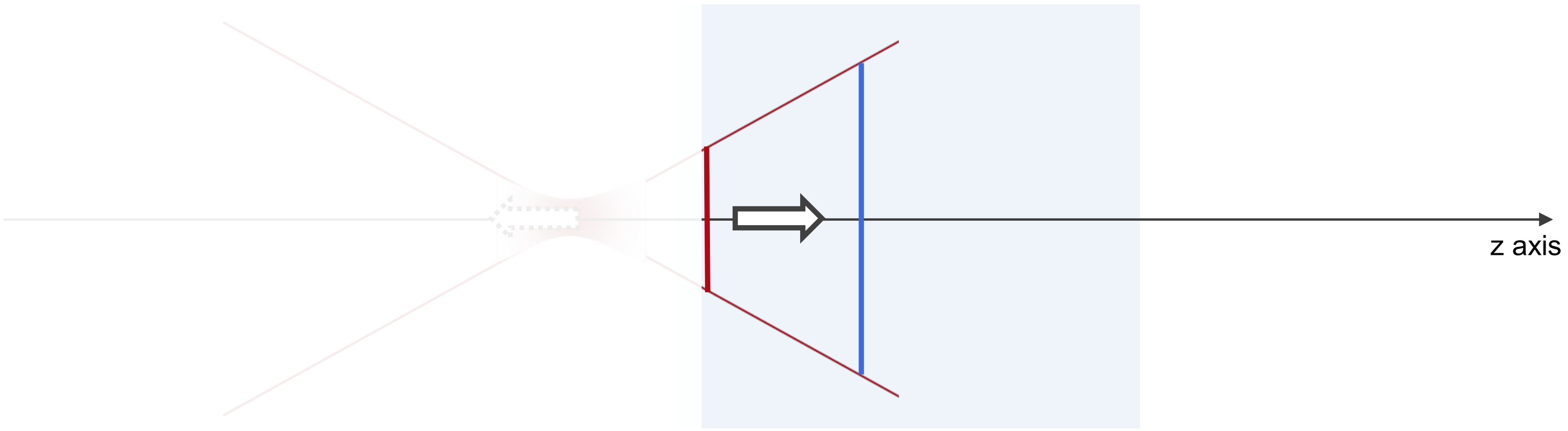
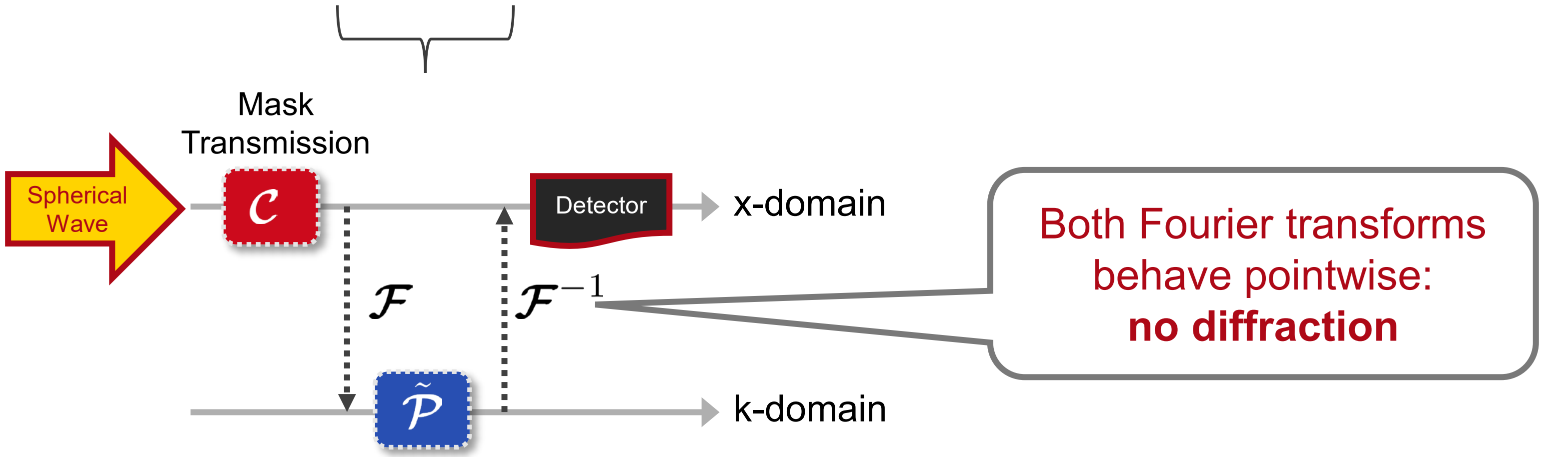
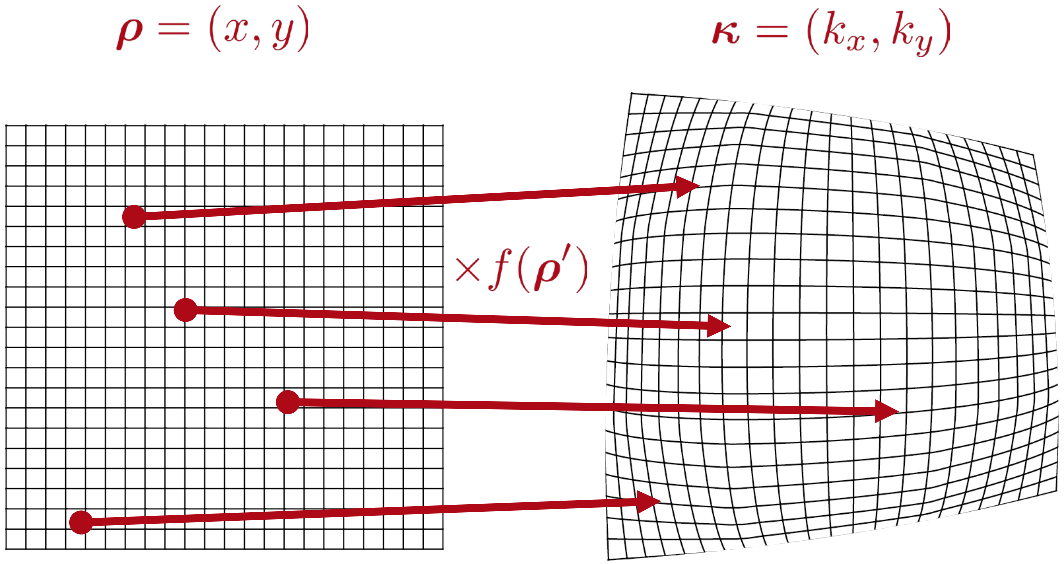
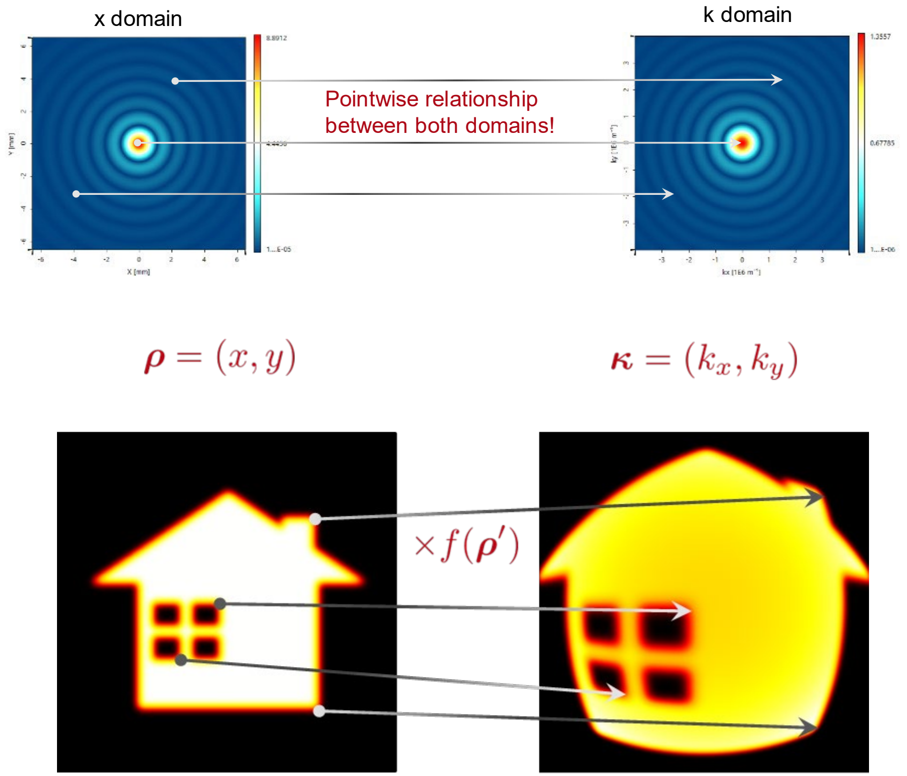




Pointwise Behavior of Fourier Transform





Pointwise Fourier Transform (PFT) Algorithm

If Fourier transform (FT) behaves approximately pointwise,

Pointwise Fourier Transform (PFT) algorithm enables very fast evaluation of FT.

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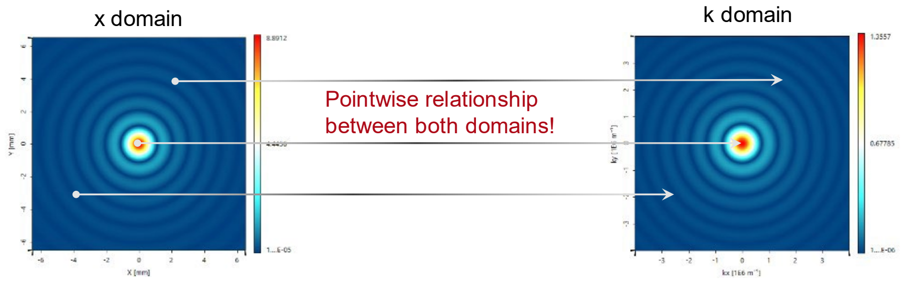
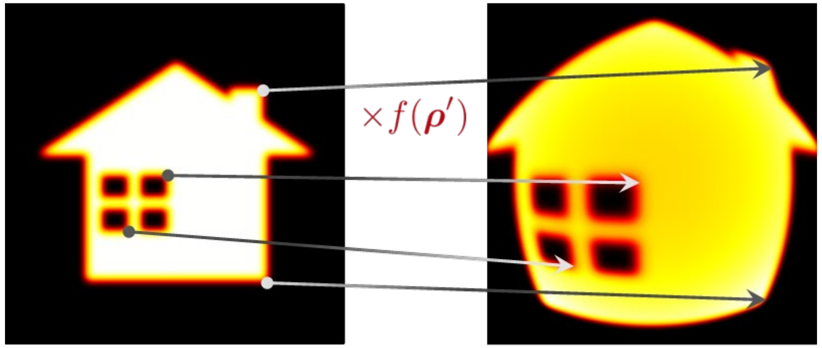
Theory and algorithm of the homeomorphic Fourier transform for optical simulations

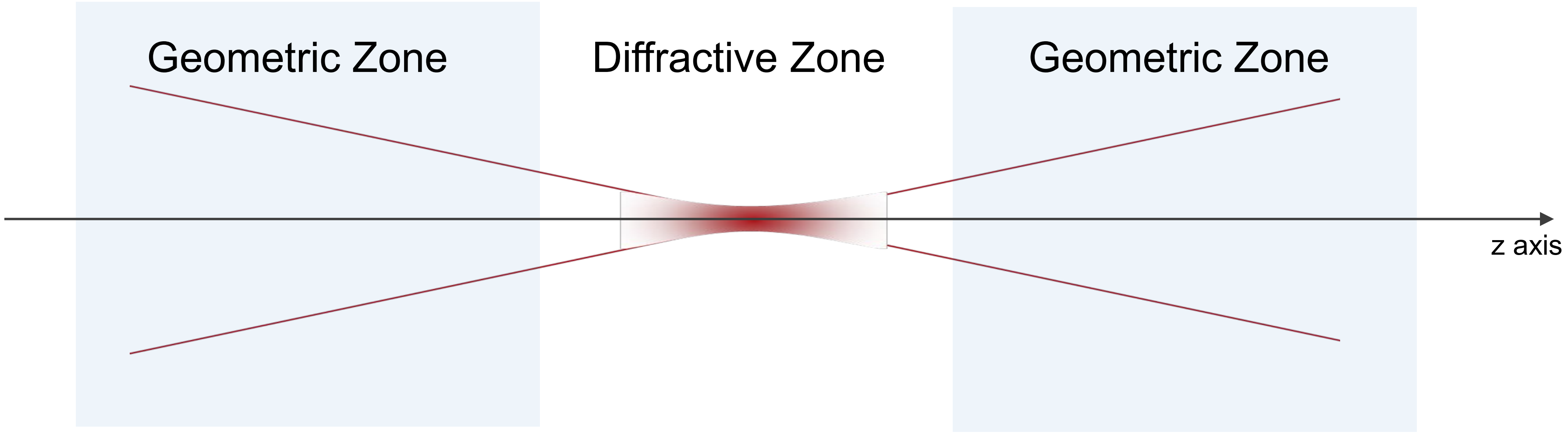
ZONGZHAO WANG,[1,2,*] OLGA BALADRON-ZORITA,[1,2] CHRISTIAN HELLMANN,[3] AND FRANK WYROWSKI[1]



$$\boldsymbol{\rho} = (x, y) \qquad \boldsymbol{\kappa} = (k_x, k_y)$$



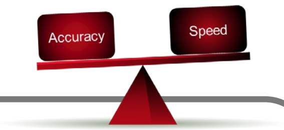
Geometric and Diffractive Field Zone



Geometric Field Zone: Fourier transform (FT) behaves approximately pointwise.

⬇

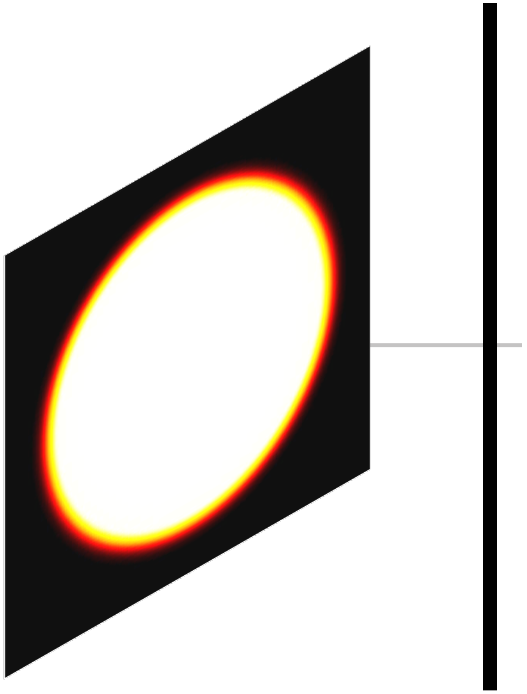
Pointwise Fourier Transform (PFT) algorithm enables very fast evaluation of FT.



Pointwise Transformation Index (PTI) used to specify threshold.

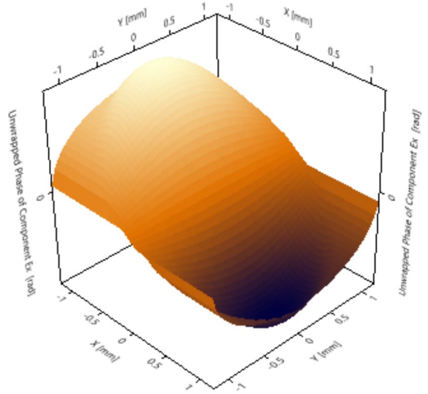
Pointwise Behavior in Presence of Aberrations

$$V_\ell(\boldsymbol{\rho}) = |V_\ell(\boldsymbol{\rho})| \exp(\mathrm{i}\psi(\boldsymbol{\rho}))$$

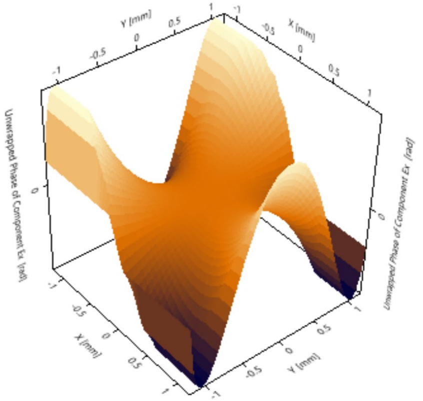


Zernike Phase Plate

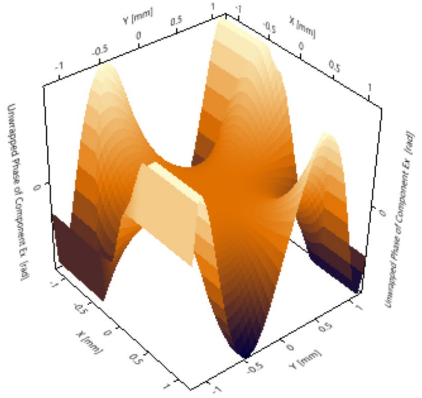
$$\psi(\boldsymbol{\rho}) = \psi^{\mathsf{sph}}(\boldsymbol{\rho}) + \psi^{\mathsf{zer}}(\boldsymbol{\rho})$$



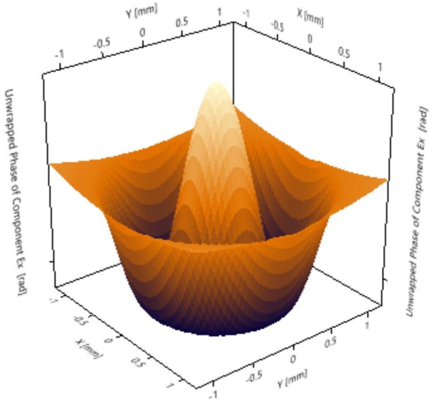
“+” coma x



“+” trefoil x

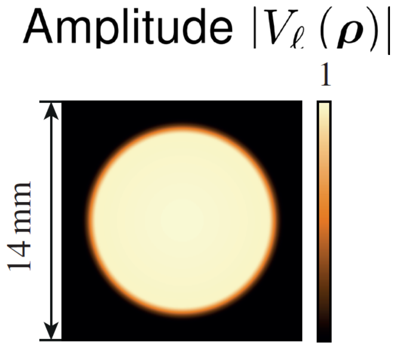


“+” tetrafoil y



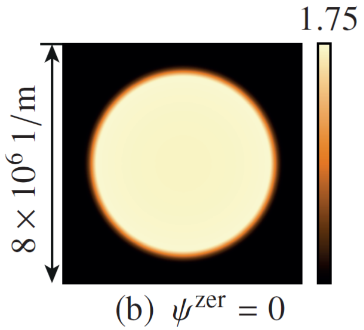
“+” tertiary spherical

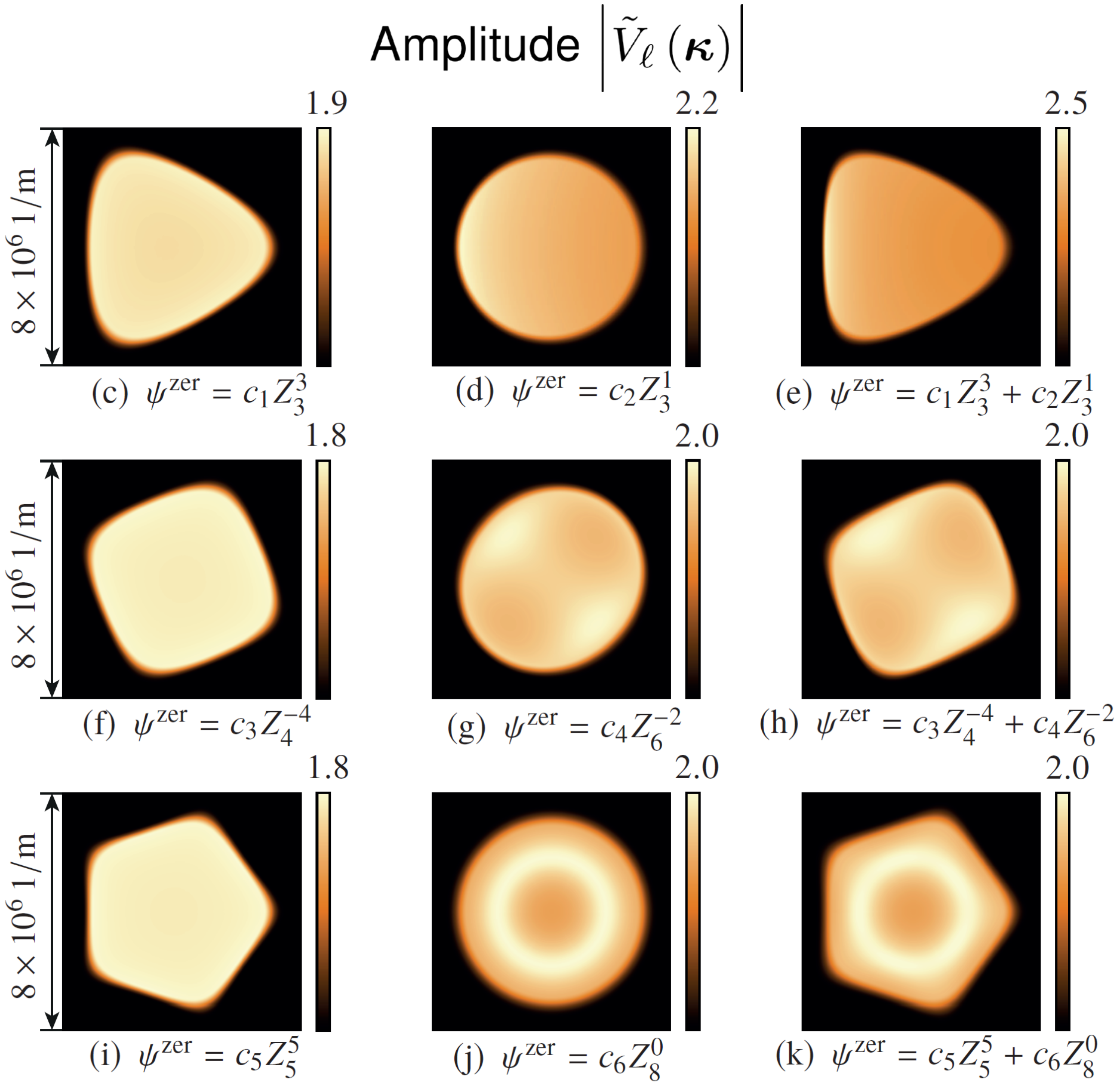
Results of Fourier Transform



$\mathcal{F}_\kappa$ →

deviation of PFT from FFT algorithms ~ 0.1%

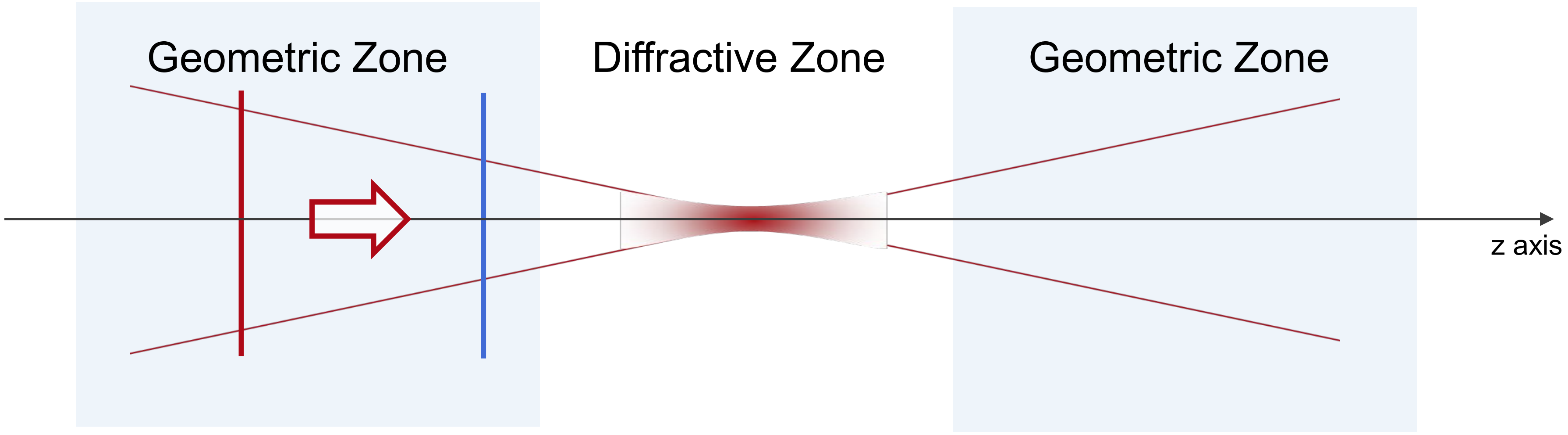


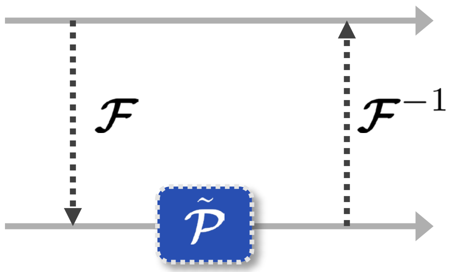


$$V_\ell(\boldsymbol{\rho})=|V_\ell(\boldsymbol{\rho})|\exp\left(\mathrm{i}\psi(\boldsymbol{\rho})\right)$$

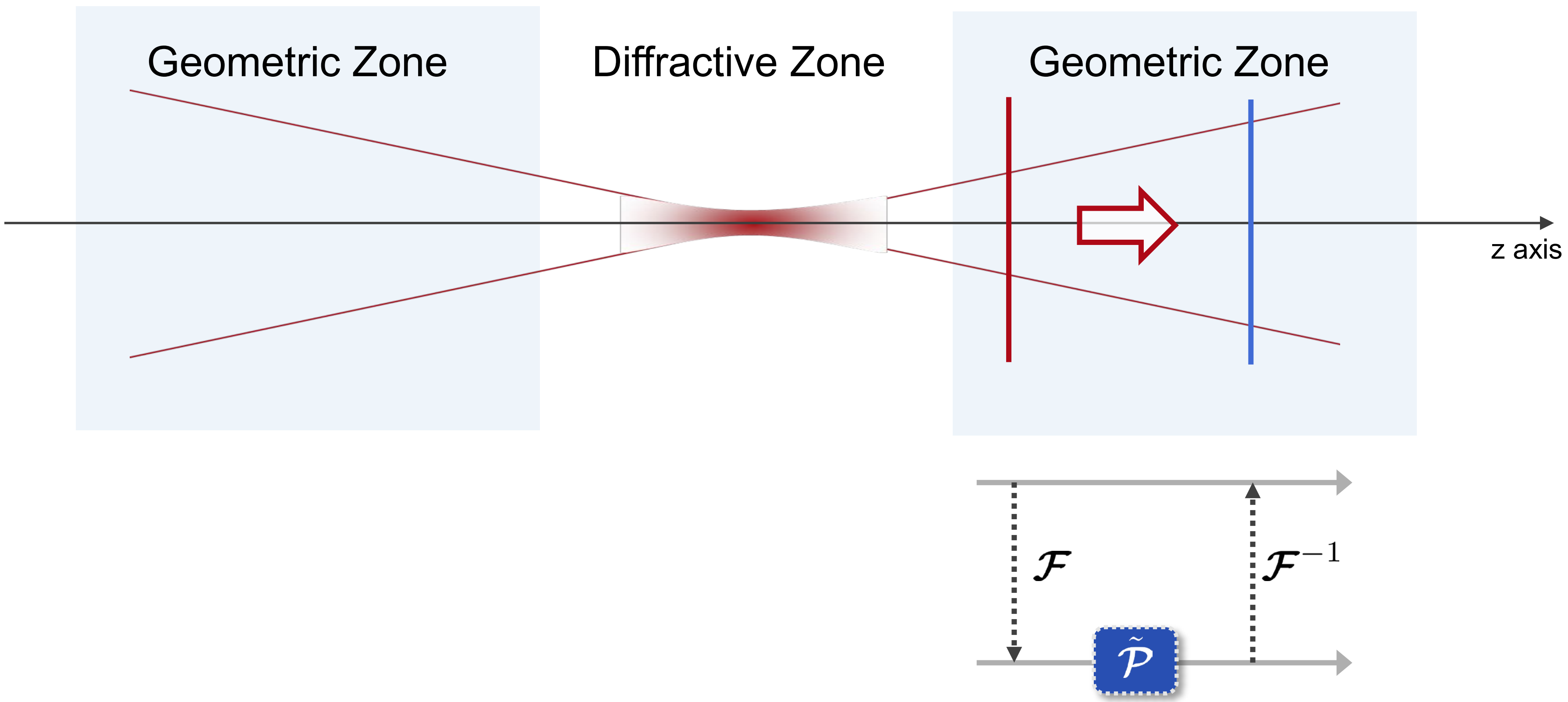
$$\psi(\boldsymbol{\rho})=\psi^{\mathsf{sph}}(\boldsymbol{\rho})+\psi^{\mathsf{zer}}(\boldsymbol{\rho})$$

Geometric Propagation of Electromagnetic Fields

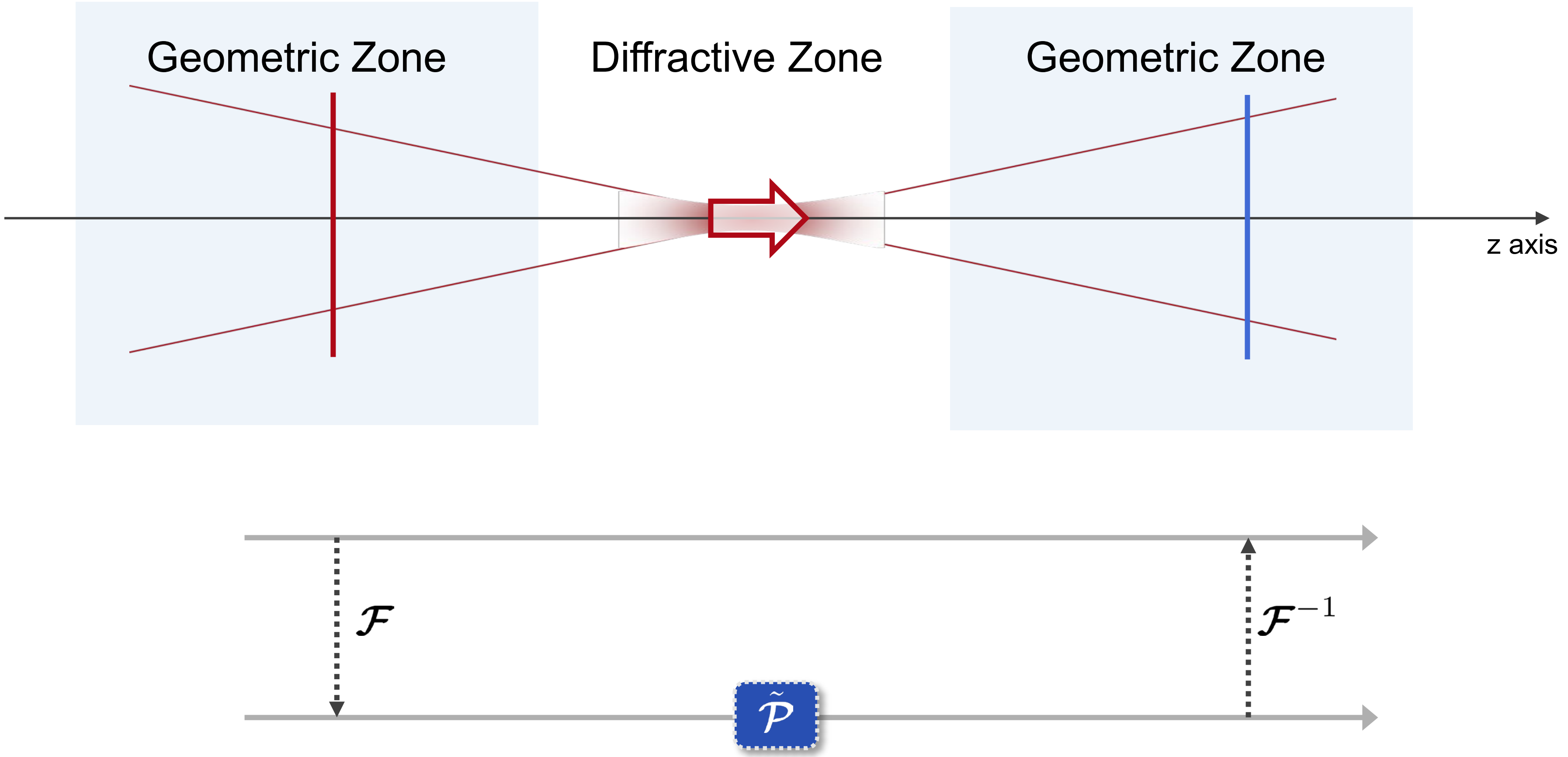




Geometric Propagation of Electromagnetic Fields



Geometric Propagation of Electromagnetic Fields



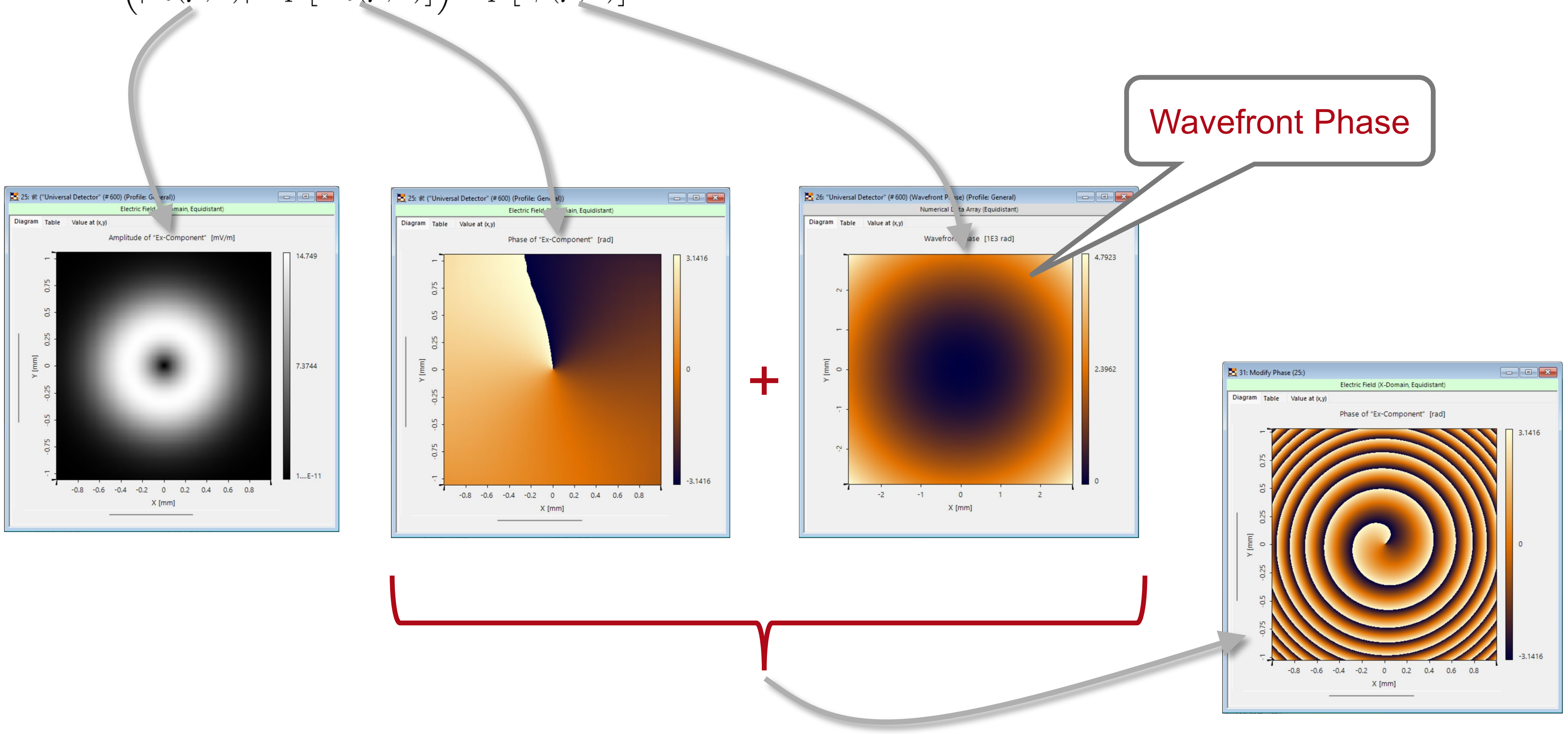
Pointwise Approximation of Fourier Transform

$$
\begin{aligned}
V_\ell(\boldsymbol{\rho}, z) &= U_\ell(\boldsymbol{\rho}, z) \exp\left[\mathrm{i}\psi(\boldsymbol{\rho}, z)\right] \\
&= \Big(|V_\ell(\boldsymbol{\rho}, z)| \exp\left[\mathrm{i}\alpha_\ell(\boldsymbol{\rho}, z)\right]\Big) \exp\left[\mathrm{i}\psi(\boldsymbol{\rho}, z)\right]
\end{aligned}
$$

Example: propagated Gaussian Laguerre beam

Pointwise Approximation of Fourier Transform

$$
\begin{aligned}
V_\ell(\boldsymbol{\rho}, z) &= U_\ell(\boldsymbol{\rho}, z)\exp\left[\mathrm{i}\psi(\boldsymbol{\rho}, z)\right] \\
&= \Big(|V_\ell(\boldsymbol{\rho}, z)|\exp\left[\mathrm{i}\alpha_\ell(\boldsymbol{\rho}, z)\right]\Big)\exp\left[\mathrm{i}\psi(\boldsymbol{\rho}, z)\right]
\end{aligned}
$$



Pointwise Approximation of Fourier Transform

$$
\begin{aligned}
V_\ell(\boldsymbol{\rho}, z) &= U_\ell(\boldsymbol{\rho}, z) \exp\left[\mathrm{i}\psi(\boldsymbol{\rho}, z)\right] \\
&= \Big(|V_\ell(\boldsymbol{\rho}, z)| \exp\left[\mathrm{i}\alpha_\ell(\boldsymbol{\rho}, z)\right]\Big) \exp\left[\mathrm{i}\psi(\boldsymbol{\rho}, z)\right]
\end{aligned}
$$

$\mathcal{F}$

$\boldsymbol{\rho} \mapsto \boldsymbol{\kappa}(\boldsymbol{\rho}) = \nabla\psi(\boldsymbol{\rho})$

$$
\begin{aligned}
\tilde{V}_\ell(\boldsymbol{\kappa}, z) &= \tilde{A}_\ell(\boldsymbol{\kappa}, z) \exp\left[\mathrm{i}\tilde{\phi}(\boldsymbol{\kappa}, z)\right] \\
&= \Big(|\tilde{V}_\ell(\boldsymbol{\kappa}, z)| \exp\left[\mathrm{i}\tilde{\alpha}_\ell(\boldsymbol{\kappa}, z)\right]\Big) \exp\left[\mathrm{i}\tilde{\phi}(\boldsymbol{\kappa}, z)\right] \\
\tilde{A}_\ell(\boldsymbol{\kappa}, z) &= \sqrt{|\mathbf{J}_\rho(\boldsymbol{\kappa})|}\Big(U_\ell(\boldsymbol{\rho}, z)\sigma(\boldsymbol{\rho}, z)\Big)[\boldsymbol{\rho} \leftarrow \boldsymbol{\rho}(\boldsymbol{\kappa})] \\
\tilde{\phi}(\boldsymbol{\kappa}, z) &= \Big(\psi(\boldsymbol{\rho}, z) - \boldsymbol{\kappa}\cdot\boldsymbol{\rho}\Big)[\boldsymbol{\rho} \leftarrow \boldsymbol{\rho}(\boldsymbol{\kappa})]
\end{aligned}
$$

Jacobian determinant

Legendre transformation

Pointwise Approximation of Fourier Transform

$$V_\ell(\boldsymbol{\rho}, z) = U_\ell(\boldsymbol{\rho}, z) \exp\left[\mathrm{i}\psi(\boldsymbol{\rho}, z)\right]$$
$$= \Big(|V_\ell(\boldsymbol{\rho}, z)| \exp\left[\mathrm{i}\alpha_\ell(\boldsymbol{\rho}, z)\right]\Big) \exp\left[\mathrm{i}\psi(\boldsymbol{\rho}, z)\right]$$

$\mathcal{F}$

$\boldsymbol{\rho} \mapsto \boldsymbol{\kappa}(\boldsymbol{\rho}) = \nabla\psi(\boldsymbol{\rho})$

$$\tilde{V}_\ell(\boldsymbol{\kappa}, z) = \tilde{A}_\ell(\boldsymbol{\kappa}, z) \exp\left[\mathrm{i}\tilde{\phi}(\boldsymbol{\kappa}, z)\right]$$
$$= \Big(|\tilde{V}_\ell(\boldsymbol{\kappa}, z)| \exp\left[\mathrm{i}\tilde{\beta}_\ell(\boldsymbol{\kappa}, z)\right]\Big) \exp\left[\mathrm{i}\tilde{\phi}(\boldsymbol{\kappa}, z)\right]$$
$$\tilde{A}_\ell(\boldsymbol{\kappa}, z) = \sqrt{|\mathbf{J}_\rho(\boldsymbol{\kappa})|}\Big(U_\ell(\boldsymbol{\rho}, z)\,\sigma(\boldsymbol{\rho}, z)\Big)[\boldsymbol{\rho} \leftarrow \boldsymbol{\rho}(\boldsymbol{\kappa})]$$
$$\tilde{\phi}(\boldsymbol{\kappa}, z) = \Big(\psi(\boldsymbol{\rho}, z) - \boldsymbol{\kappa}\cdot\boldsymbol{\rho}\Big)[\boldsymbol{\rho} \leftarrow \boldsymbol{\rho}(\boldsymbol{\kappa})]$$

$\partial_{xx}\psi(\boldsymbol{\rho}), \partial_{yy}\psi(\boldsymbol{\rho})$	$\partial_{xy}\psi(\boldsymbol{\rho})$	$\lvert\mathbf{J}_\kappa(\boldsymbol{\rho})\rvert$	factor $\sigma(\boldsymbol{\rho})$
same sign (±)	small	> 0	±i
one/both = 0	$\neq 0$	< 0	1
different sign	any	< 0	1
same sign	large	< 0	1

Pointwise Approximation of Fourier Transform

$$V_\ell(\boldsymbol{\rho}, z) = U_\ell(\boldsymbol{\rho}, z) \exp\left[\mathrm{i}\psi(\boldsymbol{\rho}, z)\right]$$
$$= \left(|V_\ell(\boldsymbol{\rho}, z)| \exp\left[\mathrm{i}\alpha_\ell(\boldsymbol{\rho}, z)\right]\right) \boxed{\exp\left[\mathrm{i}\psi(\boldsymbol{\rho}, z)\right]}$$

Exponential function must not be sampled!

$\mathcal{F}$

$\boldsymbol{\rho} \mapsto \boxed{\boldsymbol{\kappa}(\boldsymbol{\rho}) = \nabla\psi(\boldsymbol{\rho})}$

Spline interpolation of wavefront phase is suitable.

$$\tilde{V}_\ell(\boldsymbol{\kappa}, z) = \tilde{A}_\ell(\boldsymbol{\kappa}, z) \exp\left[\mathrm{i}\tilde{\phi}(\boldsymbol{\kappa}, z)\right]$$
$$= \left(|\tilde{V}_\ell(\boldsymbol{\kappa}, z)| \exp\left[\mathrm{i}\tilde{\beta}_\ell(\boldsymbol{\kappa}, z)\right]\right) \exp\left[\mathrm{i}\tilde{\phi}(\boldsymbol{\kappa}, z)\right]$$
$$\tilde{A}_\ell(\boldsymbol{\kappa}, z) = \sqrt{|\mathbf{J}_\rho(\boldsymbol{\kappa})|}\left(U_\ell(\boldsymbol{\rho}, z)\sigma(\boldsymbol{\rho}, z)\right)[\boldsymbol{\rho} \leftarrow \boldsymbol{\rho}(\boldsymbol{\kappa})]$$
$$\boxed{\tilde{\phi}(\boldsymbol{\kappa}, z) = \left(\psi(\boldsymbol{\rho}, z) - \boldsymbol{\kappa} \cdot \boldsymbol{\rho}\right)[\boldsymbol{\rho} \leftarrow \boldsymbol{\rho}(\boldsymbol{\kappa})]}$$

Spline interpolation of wavefront phase is suitable.

- Spline interpolation requires only a **small number** N of sampled values.
- The computational complexity of the Pointwise Fourier Transform algorithm scales linearly with this small N.

Pointwise Approximation of Fourier Transform

$$V_\ell(\boldsymbol{\rho}, z) = U_\ell(\boldsymbol{\rho}, z) \exp\left[\mathrm{i}\psi(\boldsymbol{\rho}, z)\right]$$
$$= \left(|V_\ell(\boldsymbol{\rho}, z)| \exp\left[\mathrm{i}\alpha_\ell(\boldsymbol{\rho}, z)\right]\right) \exp\left[\mathrm{i}\psi(\boldsymbol{\rho}, z)\right]$$

Spline interpolation of wavefront phase.

Nyquist sampling

$\mathcal{F}$

$\boldsymbol{\rho} \mapsto \boldsymbol{\kappa}(\boldsymbol{\rho}) = \nabla\psi(\boldsymbol{\rho})$

Nyquist sampling

$$\tilde{V}_\ell(\boldsymbol{\kappa}, z) = \tilde{A}_\ell(\boldsymbol{\kappa}, z) \exp\left[\mathrm{i}\tilde{\phi}(\boldsymbol{\kappa}, z)\right]$$
$$= \left(|\tilde{V}_\ell(\boldsymbol{\kappa}, z)| \exp\left[\mathrm{i}\tilde{\beta}_\ell(\boldsymbol{\kappa}, z)\right]\right) \exp\left[\mathrm{i}\tilde{\phi}(\boldsymbol{\kappa}, z)\right]$$
$$\tilde{A}_\ell(\boldsymbol{\kappa}, z) = \sqrt{|\mathbf{J}_\rho(\boldsymbol{\kappa})|}\left(U_\ell(\boldsymbol{\rho}, z)\sigma(\boldsymbol{\rho}, z)\right)[\boldsymbol{\rho} \leftarrow \boldsymbol{\rho}(\boldsymbol{\kappa})]$$
$$\tilde{\phi}(\boldsymbol{\kappa}, z) = \left(\psi(\boldsymbol{\rho}, z) - \boldsymbol{\kappa} \cdot \boldsymbol{\rho}\right)[\boldsymbol{\rho} \leftarrow \boldsymbol{\rho}(\boldsymbol{\kappa})]$$

Spline interpolation of wavefront phase.

The pointwise Fourier transform algorithm requires incorporating both **gridded and gridless sampling techniques**.

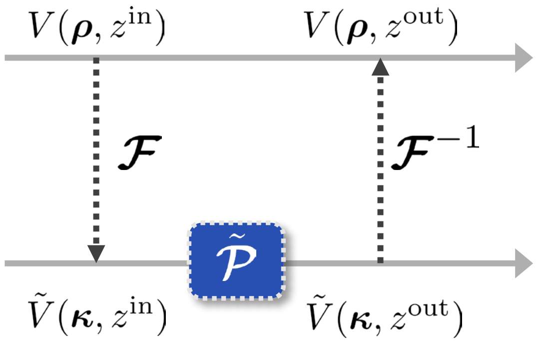
Algorithm for Geometric Propagation of Electromagnetic Fields

$$V_\ell(\boldsymbol{\rho},z) = U_\ell(\boldsymbol{\rho},z)\exp\left[\mathrm{i}\psi(\boldsymbol{\rho},z)\right]$$
$$= \left(|V_\ell(\boldsymbol{\rho},z)|\exp\left[\mathrm{i}\alpha_\ell(\boldsymbol{\rho},z)\right]\right)\exp\left[\mathrm{i}\psi(\boldsymbol{\rho},z)\right]$$

$\boldsymbol{\mathcal{F}}$

$\boldsymbol{\rho} \mapsto \boldsymbol{\kappa}(\boldsymbol{\rho}) = \nabla\psi(\boldsymbol{\rho})$

$$\tilde{V}_\ell(\boldsymbol{\kappa},z) = \tilde{A}_\ell(\boldsymbol{\kappa},z)\exp\left[\mathrm{i}\tilde{\phi}(\boldsymbol{\kappa},z)\right]$$
$$= \left(|\tilde{V}_\ell(\boldsymbol{\kappa},z)|\exp\left[\mathrm{i}\tilde{\beta}_\ell(\boldsymbol{\kappa},z)\right]\right)\exp\left[\mathrm{i}\tilde{\phi}(\boldsymbol{\kappa},z)\right]$$
$$\tilde{A}_\ell(\boldsymbol{\kappa},z) = \sqrt{|\mathbf{J}_\rho(\boldsymbol{\kappa})|}\left(U_\ell(\boldsymbol{\rho},z)\sigma(\boldsymbol{\rho},z)\right)[\boldsymbol{\rho}\leftarrow\boldsymbol{\rho}(\boldsymbol{\kappa})]$$
$$\tilde{\phi}(\boldsymbol{\kappa},z) = \left(\psi(\boldsymbol{\rho},z) - \boldsymbol{\kappa}\cdot\boldsymbol{\rho}\right)[\boldsymbol{\rho}\leftarrow\boldsymbol{\rho}(\boldsymbol{\kappa})]$$



$\times \exp\left[\mathrm{i}\check{k}_z\Delta z\right]$

$$V_\ell(\boldsymbol{\rho},z) = U_\ell(\boldsymbol{\rho},z)\exp\left[\mathrm{i}\psi(\boldsymbol{\rho},z)\right]$$
$$= \left(|V_\ell(\boldsymbol{\rho},z)|\exp\left[\mathrm{i}\alpha_\ell(\boldsymbol{\rho},z)\right]\right)\exp\left[\mathrm{i}\psi(\boldsymbol{\rho},z)\right]$$
$$U_\ell(\boldsymbol{\rho},z) = \sqrt{|\mathbf{J}_k(\boldsymbol{\rho})|}\left(\tilde{A}_\ell(\boldsymbol{\kappa},z)\tilde{\sigma}(\boldsymbol{\kappa},z)\right)[\boldsymbol{\kappa}\leftarrow\boldsymbol{\kappa}(\boldsymbol{\rho})]$$
$$\psi(\boldsymbol{\rho},z) = \left(\tilde{\phi}(\boldsymbol{\kappa},z) + \boldsymbol{\kappa}\cdot\boldsymbol{\rho}\right)[\boldsymbol{\kappa}\leftarrow\boldsymbol{\kappa}(\boldsymbol{\rho})]$$

$\boldsymbol{\mathcal{F}}^{-1}$

$\boldsymbol{\kappa} \mapsto \boldsymbol{\rho}(\boldsymbol{\kappa}) = -\tilde{\nabla}\tilde{\phi}(\boldsymbol{\kappa})$

$$\tilde{V}_\ell(\boldsymbol{\kappa},z) = \tilde{A}_\ell(\boldsymbol{\kappa},z)\exp\left[\mathrm{i}\tilde{\phi}(\boldsymbol{\kappa},z)\right]$$
$$= \left(|\tilde{V}_\ell(\boldsymbol{\kappa},z)|\exp\left[\mathrm{i}\tilde{\beta}_\ell(\boldsymbol{\kappa},z)\right]\right)\exp\left[\mathrm{i}\tilde{\phi}(\boldsymbol{\kappa},z)\right]$$

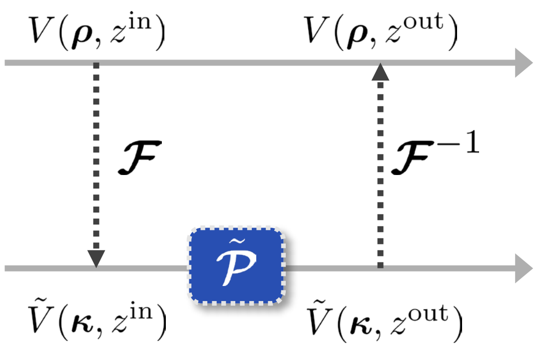
Pointwise Approximation of Fourier Transform

$$V_\ell(\boldsymbol{\rho}, z) = U_\ell(\boldsymbol{\rho}, z) \exp\left[\mathrm{i}\psi(\boldsymbol{\rho}, z)\right]$$
$$= \Big(|V_\ell(\boldsymbol{\rho}, z)| \exp\left[\mathrm{i}\alpha_\ell(\boldsymbol{\rho}, z)\right]\Big) \exp\left[\mathrm{i}\psi(\boldsymbol{\rho}, z)\right]$$

$\boldsymbol{\mathcal{F}}$

$\boldsymbol{\rho} \mapsto \boldsymbol{\kappa}(\boldsymbol{\rho}) = \nabla\psi(\boldsymbol{\rho})$

$$\tilde{V}_\ell(\boldsymbol{\kappa}, z) = \tilde{A}_\ell(\boldsymbol{\kappa}, z) \exp\left[\mathrm{i}\tilde{\phi}(\boldsymbol{\kappa}, z)\right]$$
$$= \Big(|\tilde{V}_\ell(\boldsymbol{\kappa}, z)| \exp\left[\mathrm{i}\tilde{\beta}_\ell(\boldsymbol{\kappa}, z)\right]\Big) \exp\left[\mathrm{i}\tilde{\phi}(\boldsymbol{\kappa}, z)\right]$$
$$\tilde{A}_\ell(\boldsymbol{\kappa}, z) = \sqrt{|\mathbf{J}_\rho(\boldsymbol{\kappa})|}\Big(U_\ell(\boldsymbol{\rho}, z)\boxed{\sigma(\boldsymbol{\rho}, z)}\Big)[\boldsymbol{\rho} \leftarrow \boldsymbol{\rho}(\boldsymbol{\kappa})]$$
$$\tilde{\phi}(\boldsymbol{\kappa}, z) = \Big(\psi(\boldsymbol{\rho}, z) - \boldsymbol{\kappa}\cdot\boldsymbol{\rho}\Big)[\boldsymbol{\rho} \leftarrow \boldsymbol{\rho}(\boldsymbol{\kappa})]$$



$$\times \exp\left[\mathrm{i}\check{k}_z \Delta z\right]$$

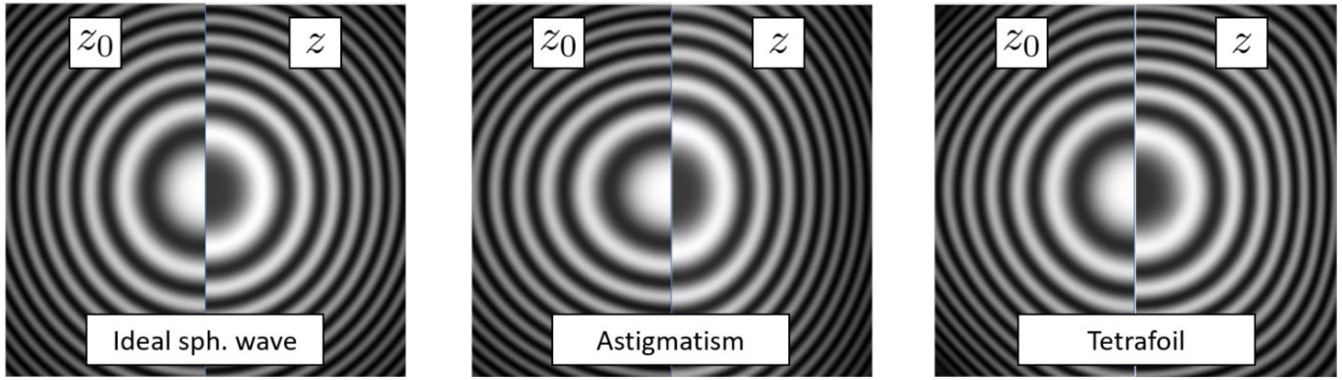
$$V_\ell(\boldsymbol{\rho}, z) = U_\ell(\boldsymbol{\rho}, z) \exp\left[\mathrm{i}\psi(\boldsymbol{\rho}, z)\right]$$
$$= \Big(|V_\ell(\boldsymbol{\rho}, z)| \exp\left[\mathrm{i}\alpha_\ell(\boldsymbol{\rho}, z)\right]\Big) \exp\left[\mathrm{i}\psi(\boldsymbol{\rho}, z)\right]$$
$$U_\ell(\boldsymbol{\rho}, z) = \sqrt{|\mathbf{J}_k(\boldsymbol{\rho})|}\Big(\tilde{A}_\ell(\boldsymbol{\kappa}, z)\boxed{\tilde{\sigma}(\boldsymbol{\kappa}, z)}\Big)[\boldsymbol{\kappa} \leftarrow \boldsymbol{\kappa}(\boldsymbol{\rho})]$$
$$\psi(\boldsymbol{\rho}, z) = \Big(\tilde{\phi}(\boldsymbol{\kappa}, z) + \boldsymbol{\kappa}\cdot\boldsymbol{\rho}\Big)[\boldsymbol{\kappa} \leftarrow \boldsymbol{\kappa}(\boldsymbol{\rho})]$$

$\boldsymbol{\mathcal{F}}^{-1}$

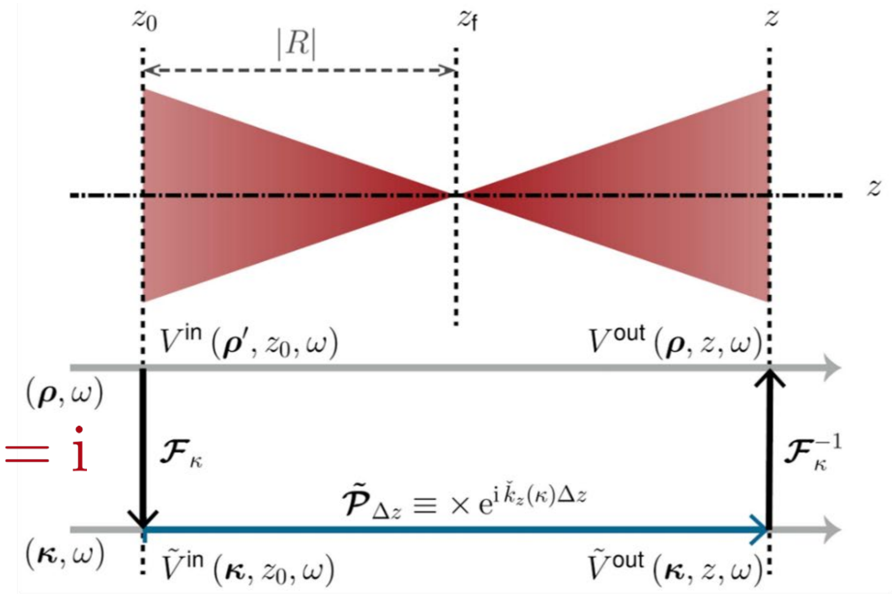
$\boldsymbol{\kappa} \mapsto \boldsymbol{\rho}(\boldsymbol{\kappa}) = -\tilde{\nabla}\tilde{\phi}(\boldsymbol{\kappa})$

$$\tilde{V}_\ell(\boldsymbol{\kappa}, z) = \tilde{A}_\ell(\boldsymbol{\kappa}, z) \exp\left[\mathrm{i}\tilde{\phi}(\boldsymbol{\kappa}, z)\right]$$
$$= \Big(|\tilde{V}_\ell(\boldsymbol{\kappa}, z)| \exp\left[\mathrm{i}\tilde{\beta}_\ell(\boldsymbol{\kappa}, z)\right]\Big) \exp\left[\mathrm{i}\tilde{\phi}(\boldsymbol{\kappa}, z)\right]$$

Gouy Phase Shift

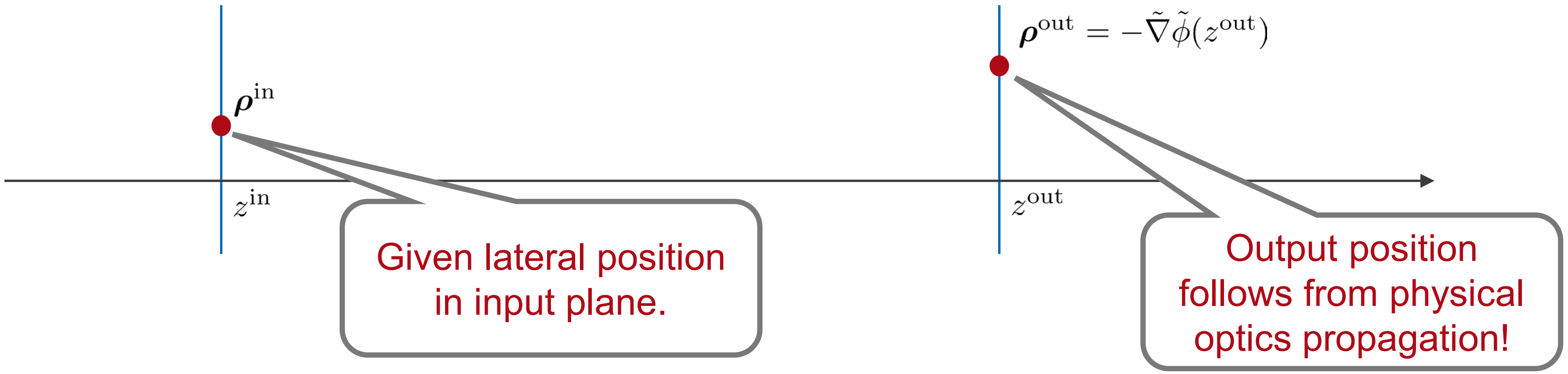


$\sigma = \mathrm{i}$



$\tilde{\sigma} = \mathrm{i}$

Analysis of Coordinate Mapping



Analysis of Coordinate Mapping

$$\times \exp\left[\mathrm{i}\check{k}_z \Delta z\right]$$

$$\begin{aligned}\tilde{\phi}(\boldsymbol{\kappa}, z^{\text{out}}) &= \tilde{\phi}(\boldsymbol{\kappa}, z^{\text{in}}) + \Delta z\, \Re \check{k}_z(\boldsymbol{\kappa}) \\ &= \tilde{\phi}(\boldsymbol{\kappa}, z^{\text{in}}) + \Delta z\, \Re\sqrt{k_0^2 \check{n}^2 - \|\boldsymbol{\kappa}\|^2}\end{aligned}$$

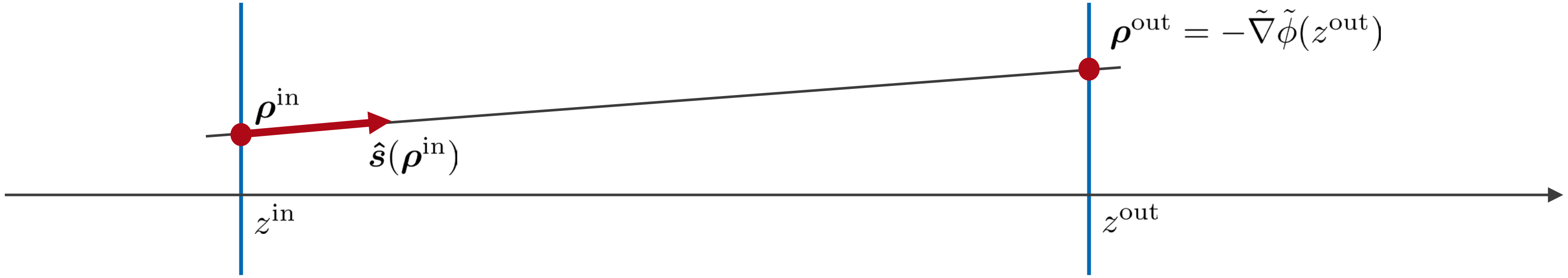
Analysis of Coordinate Mapping

$$\begin{aligned}\tilde{\phi}(\boldsymbol{\kappa}, z^{\mathrm{out}}) &= \tilde{\phi}(\boldsymbol{\kappa}, z^{\mathrm{in}}) + \Delta z \; \Re \check{k}_z(\boldsymbol{\kappa}) \\ &= \tilde{\phi}(\boldsymbol{\kappa}, z^{\mathrm{in}}) + \Delta z \; \Re\sqrt{k_0^2 \check{n}^2 - \|\boldsymbol{\kappa}\|^2}\end{aligned}$$

$$\begin{aligned}\boldsymbol{\rho}^{\mathrm{out}}(\boldsymbol{\kappa}) &= -\left(\tilde{\nabla}\tilde{\phi}(z^{\mathrm{out}})\right)(\boldsymbol{\kappa}) \\ &= \boldsymbol{\rho}^{\mathrm{in}}(\boldsymbol{\kappa}) - \Delta z \; \Re\left(\tilde{\nabla}\check{k}_z\right)(\boldsymbol{\kappa}) \\ &= \boldsymbol{\rho}^{\mathrm{in}}(\boldsymbol{\kappa}) + \Delta z \; \frac{k_z(\boldsymbol{\kappa})}{k_z^2(\boldsymbol{\kappa}) + \bar{k}_z^2(\boldsymbol{\kappa})} \; \boldsymbol{\kappa}\end{aligned}$$

$$\check{k}_z = k_z + \mathrm{i}\underline{k}_z$$

Analysis of Coordinate Mapping



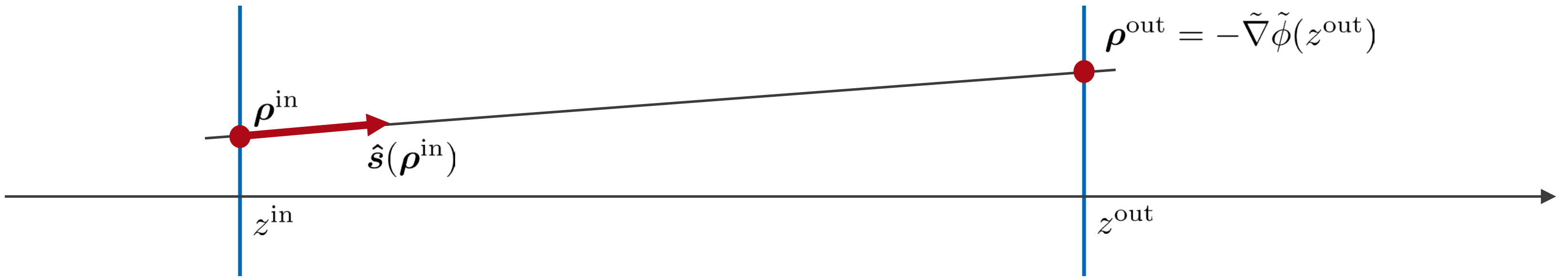
$$\hat{\boldsymbol{s}} := \frac{\boldsymbol{r}^{\mathrm{out}} - \boldsymbol{r}^{\mathrm{in}}}{\|\boldsymbol{r}^{\mathrm{out}} - \boldsymbol{r}^{\mathrm{in}}\|} = \frac{(\boldsymbol{\rho}^{\mathrm{out}}, z^{\mathrm{out}}) - (\boldsymbol{\rho}^{\mathrm{in}}, z^{\mathrm{in}})}{\|(\boldsymbol{\rho}^{\mathrm{out}}, z^{\mathrm{out}}) - (\boldsymbol{\rho}^{\mathrm{in}}, z^{\mathrm{in}})\|}$$

$$\begin{aligned}\tilde{\phi}(\boldsymbol{\kappa}, z^{\mathrm{out}}) &= \tilde{\phi}(\boldsymbol{\kappa}, z^{\mathrm{in}}) + \Delta z\ \Re \check{k}_z(\boldsymbol{\kappa}) \\ &= \tilde{\phi}(\boldsymbol{\kappa}, z^{\mathrm{in}}) + \Delta z\ \Re\sqrt{k_0^2 \check{n}^2 - \|\boldsymbol{\kappa}\|^2}\end{aligned}$$

$$\begin{aligned}\boldsymbol{\rho}^{\mathrm{out}}(\boldsymbol{\kappa}) &= -\big(\tilde{\nabla}\tilde{\phi}(z^{\mathrm{out}})\big)(\boldsymbol{\kappa}) \\ &= \boldsymbol{\rho}^{\mathrm{in}}(\boldsymbol{\kappa}) - \Delta z\ \Re\left(\tilde{\nabla}\check{k}_z\right)(\boldsymbol{\kappa}) \\ &= \boldsymbol{\rho}^{\mathrm{in}}(\boldsymbol{\kappa}) + \Delta z\ \frac{k_z(\boldsymbol{\kappa})}{k_z^2(\boldsymbol{\kappa}) + \bar{k}_z^2(\boldsymbol{\kappa})}\ \boldsymbol{\kappa}\end{aligned}$$

$$\check{k}_z = k_z + \mathrm{i}\underline{k}_z$$

Analysis of Coordinate Mapping



$$\hat{\boldsymbol{s}} := \frac{\boldsymbol{r}^{\mathrm{out}} - \boldsymbol{r}^{\mathrm{in}}}{\|\boldsymbol{r}^{\mathrm{out}} - \boldsymbol{r}^{\mathrm{in}}\|} = \frac{(\boldsymbol{\rho}^{\mathrm{out}}, z^{\mathrm{out}}) - (\boldsymbol{\rho}^{\mathrm{in}}, z^{\mathrm{in}})}{\|(\boldsymbol{\rho}^{\mathrm{out}}, z^{\mathrm{out}}) - (\boldsymbol{\rho}^{\mathrm{in}}, z^{\mathrm{in}})\|}$$

$$\hat{\boldsymbol{s}}_{\perp}(\boldsymbol{\rho}, z^{\mathrm{in}}) := \left(\frac{k_z(\boldsymbol{\kappa})\ \boldsymbol{\kappa}}{\sqrt{k_z^2(\boldsymbol{\kappa})\ \|\boldsymbol{\kappa}\|^2 + \left(k_z^2(\boldsymbol{\kappa}) + \underline{k}_z^2(\boldsymbol{\kappa})\right)^2}}\right)\left[\boldsymbol{\kappa} \leftarrow (\nabla\psi(z^{\mathrm{in}}))(\boldsymbol{\rho})\right]$$

No or very small absorption

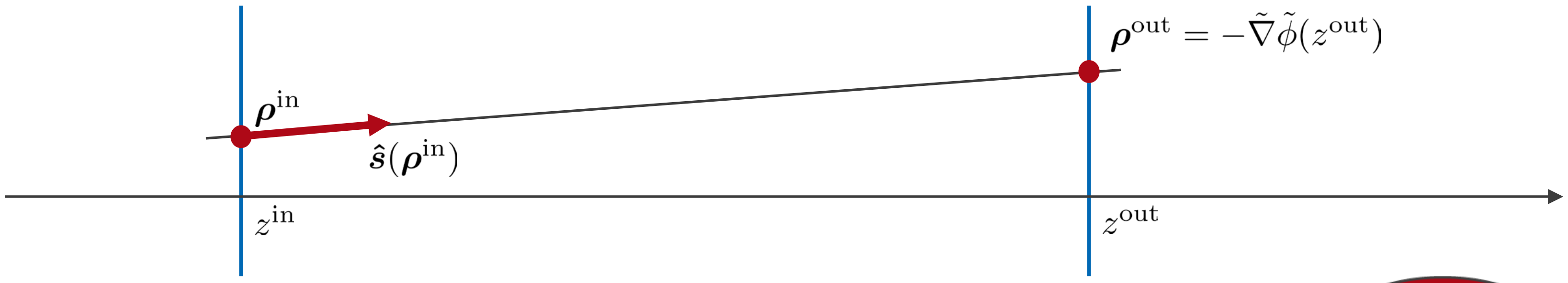
$$\hat{\boldsymbol{s}}_{\perp}(\boldsymbol{\rho}, z^{\mathrm{in}}) = \frac{(\nabla\psi(z^{\mathrm{in}}))(\boldsymbol{\rho})}{k_0 n}$$

$$\tilde{\phi}(\boldsymbol{\kappa}, z^{\mathrm{out}}) = \tilde{\phi}(\boldsymbol{\kappa}, z^{\mathrm{in}}) + \Delta z\ \Re \check{k}_z(\boldsymbol{\kappa})$$
$$= \tilde{\phi}(\boldsymbol{\kappa}, z^{\mathrm{in}}) + \Delta z\ \Re\sqrt{k_0^2 \check{n}^2 - \|\boldsymbol{\kappa}\|^2}$$

$$\boldsymbol{\rho}^{\mathrm{out}}(\boldsymbol{\kappa}) = -\left(\tilde{\nabla}\tilde{\phi}(z^{\mathrm{out}})\right)(\boldsymbol{\kappa})$$
$$= \boldsymbol{\rho}^{\mathrm{in}}(\boldsymbol{\kappa}) - \Delta z\ \Re\left(\tilde{\nabla}\check{k}_z\right)(\boldsymbol{\kappa})$$
$$= \boldsymbol{\rho}^{\mathrm{in}}(\boldsymbol{\kappa}) + \Delta z\ \frac{k_z(\boldsymbol{\kappa})}{k_z^2(\boldsymbol{\kappa}) + \underline{k}_z^2(\boldsymbol{\kappa})}\ \boldsymbol{\kappa}$$

$$\check{k}_z = k_z + \mathrm{i}\underline{k}_z$$

Analysis of Coordinate Mapping

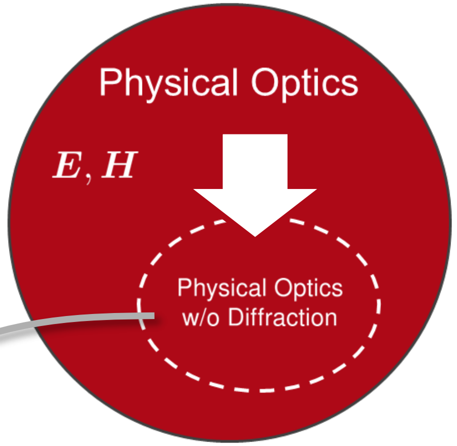


$$\hat{\boldsymbol{s}} := \frac{\boldsymbol{r}^{\text{out}} - \boldsymbol{r}^{\text{in}}}{\|\boldsymbol{r}^{\text{out}} - \boldsymbol{r}^{\text{in}}\|} = \frac{(\boldsymbol{\rho}^{\text{out}}, z^{\text{out}}) - (\boldsymbol{\rho}^{\text{in}}, z^{\text{in}})}{\|(\boldsymbol{\rho}^{\text{out}}, z^{\text{out}}) - (\boldsymbol{\rho}^{\text{in}}, z^{\text{in}})\|}$$

$$\hat{\boldsymbol{s}}_{\perp}(\boldsymbol{\rho}, z^{\text{in}}) := \left(\frac{k_z(\boldsymbol{\kappa})\, \boldsymbol{\kappa}}{\sqrt{k_z^2(\boldsymbol{\kappa})\, \|\boldsymbol{\kappa}\|^2 + \left(k_z^2(\boldsymbol{\kappa}) + \underline{k}_z^2(\boldsymbol{\kappa})\right)^2}} \right) \left[\boldsymbol{\kappa} \leftarrow (\nabla\psi(z^{\text{in}}))(\boldsymbol{\rho})\right]$$

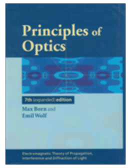
No or very small absorption

$$\hat{\boldsymbol{s}}_{\perp}(\boldsymbol{\rho}, z^{\text{in}}) := \frac{(\nabla\psi(z^{\text{in}}))(\boldsymbol{\rho})}{k_0 n}$$



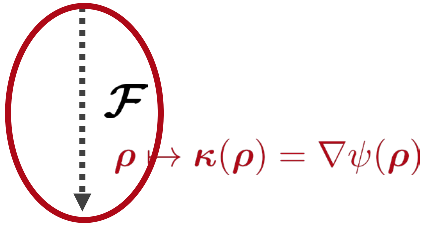
We need to identify that part of physical optics, which deals with the "***geometrical laws relating to the propagation of the 'amplitude vectors' E and H.***"

Citation
Page 125

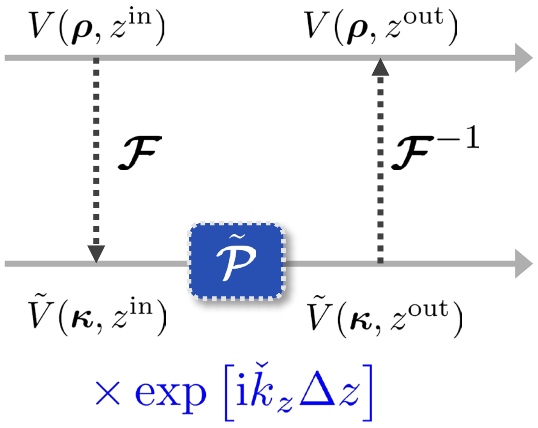


Pointwise Approximation of Fourier Transform

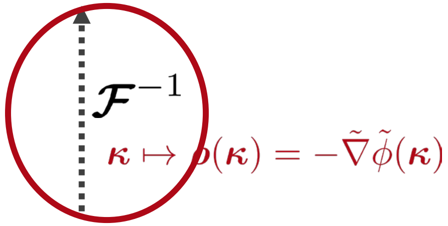
$$V_\ell(\boldsymbol{\rho}, z) = U_\ell(\boldsymbol{\rho}, z)\exp\left[\mathrm{i}\psi(\boldsymbol{\rho}, z)\right]$$
$$= \left(|V_\ell(\boldsymbol{\rho}, z)|\exp\left[\mathrm{i}\alpha_\ell(\boldsymbol{\rho}, z)\right]\right)\exp\left[\mathrm{i}\psi(\boldsymbol{\rho}, z)\right]$$



$$\tilde V_\ell(\boldsymbol{\kappa}, z) = \tilde A_\ell(\boldsymbol{\kappa}, z)\exp\left[\mathrm{i}\tilde\phi(\boldsymbol{\kappa}, z)\right]$$
$$= \left(|\tilde V_\ell(\boldsymbol{\kappa}, z)|\exp\left[\mathrm{i}\tilde\beta_\ell(\boldsymbol{\kappa}, z)\right]\right)\exp\left[\mathrm{i}\tilde\phi(\boldsymbol{\kappa}, z)\right]$$
$$\tilde A_\ell(\boldsymbol{\kappa}, z) = \sqrt{|\mathbf{J}_\rho(\boldsymbol{\kappa})|}\left(U_\ell(\boldsymbol{\rho}, z)\,\sigma(\boldsymbol{\rho}, z)\right)[\boldsymbol{\rho} \leftarrow \boldsymbol{\rho}(\boldsymbol{\kappa})]$$



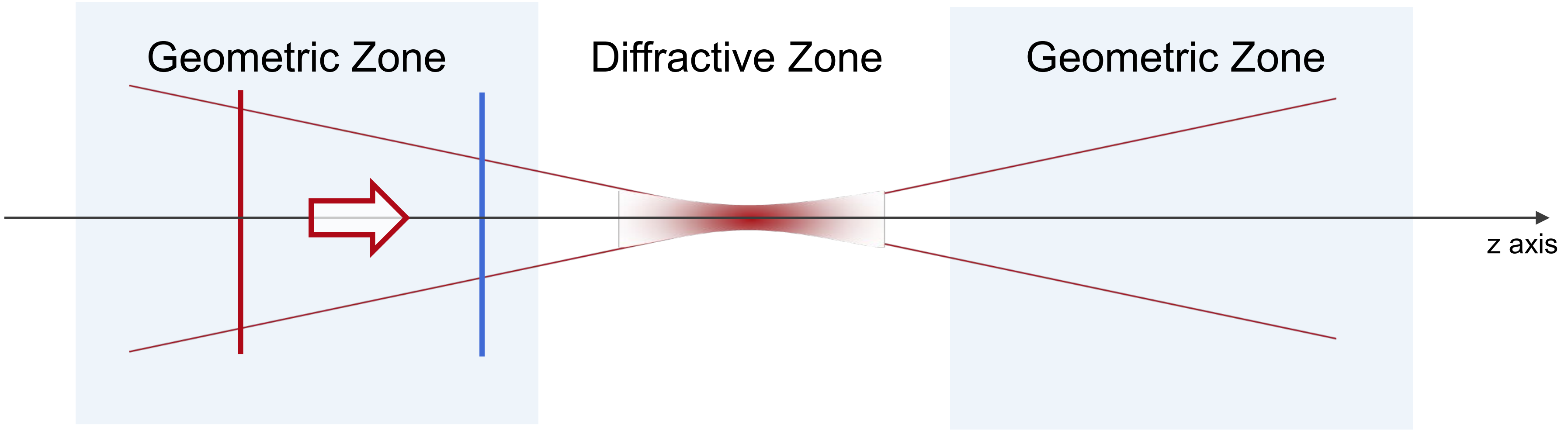
$$V_\ell(\boldsymbol{\rho}, z) = U_\ell(\boldsymbol{\rho}, z)\exp\left[\mathrm{i}\psi(\boldsymbol{\rho}, z)\right]$$
$$= \left(|V_\ell(\boldsymbol{\rho}, z)|\exp\left[\mathrm{i}\alpha_\ell(\boldsymbol{\rho}, z)\right]\right)\exp\left[\mathrm{i}\psi(\boldsymbol{\rho}, z)\right]$$
$$U_\ell(\boldsymbol{\rho}, z) = \sqrt{|\mathbf{J}_k(\boldsymbol{\rho})|}\left(\tilde A_\ell(\boldsymbol{\kappa}, z)\,\tilde\sigma(\boldsymbol{\kappa}, z)\right)[\boldsymbol{\kappa} \leftarrow \boldsymbol{\kappa}(\boldsymbol{\rho})]$$
$$\psi(\boldsymbol{\rho}, z) = \left(\tilde\phi(\boldsymbol{\kappa}, z) + \boldsymbol{\kappa}\cdot\boldsymbol{\rho}\right)[\boldsymbol{\kappa} \leftarrow \boldsymbol{\kappa}(\boldsymbol{\rho})]$$

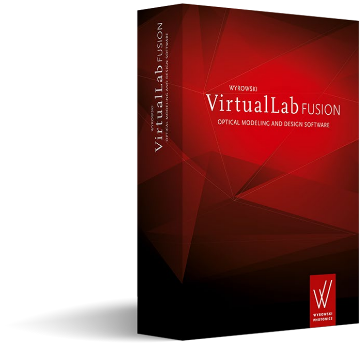


$$\tilde V_\ell(\boldsymbol{\kappa}, z) = \tilde A_\ell(\boldsymbol{\kappa}, z)\exp\left[\mathrm{i}\tilde\phi(\boldsymbol{\kappa}, z)\right]$$

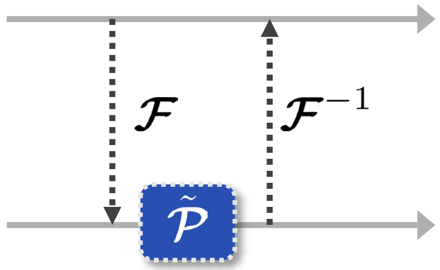
Seamless inclusion of diffraction by replacing one or both PFT by FT!

Propagation Scenarios and Selection of Fourier Transform





Geometric Propagation



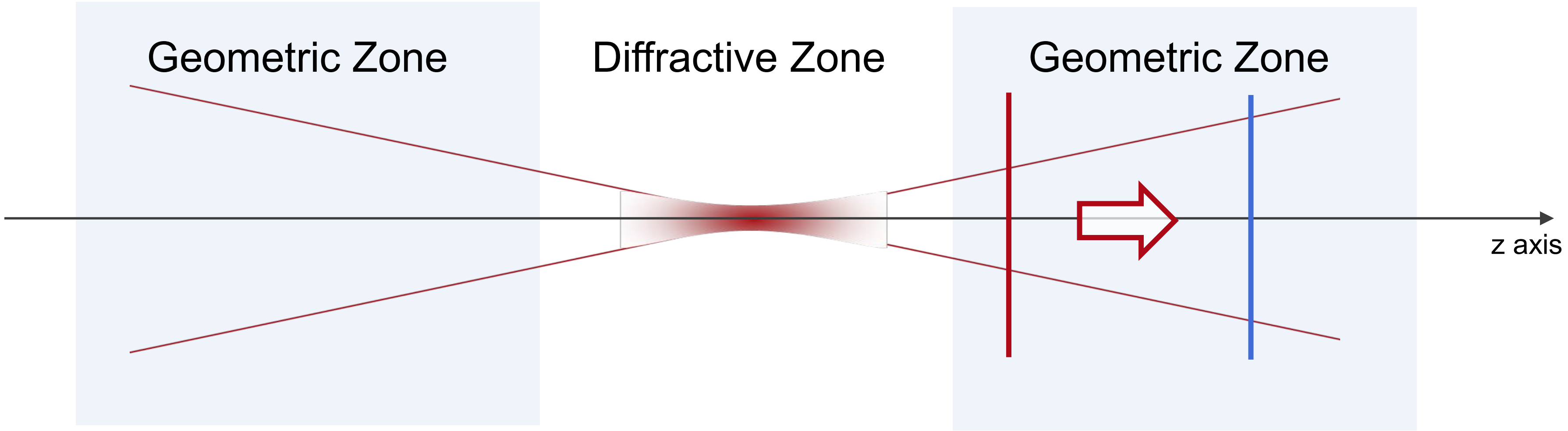
2019

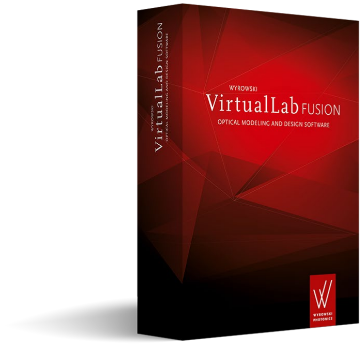
Research Article Vol. 36, No. 9 / September 2019 / Journal of the Optical Society of America A 1551
Journal of the Optical Society of America A OPTICS, IMAGE SCIENCE, AND VISION

Isolating the Gouy phase shift in a full physical-optics solution to the propagation problem

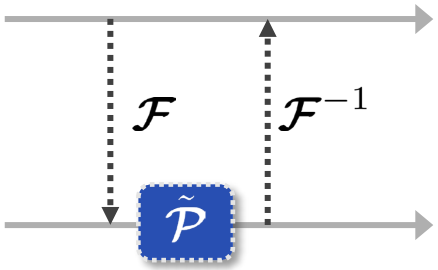
Olga Baladron-Zorita,[1,2,*] Zongzhao Wang,[1,2] Christian Hellmann,[2,3] and Frank Wyrowski[1]

Propagation Scenarios and Selection of Fourier Transform





Geometric Propagation



2019

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Journal of the Optical Society of America A

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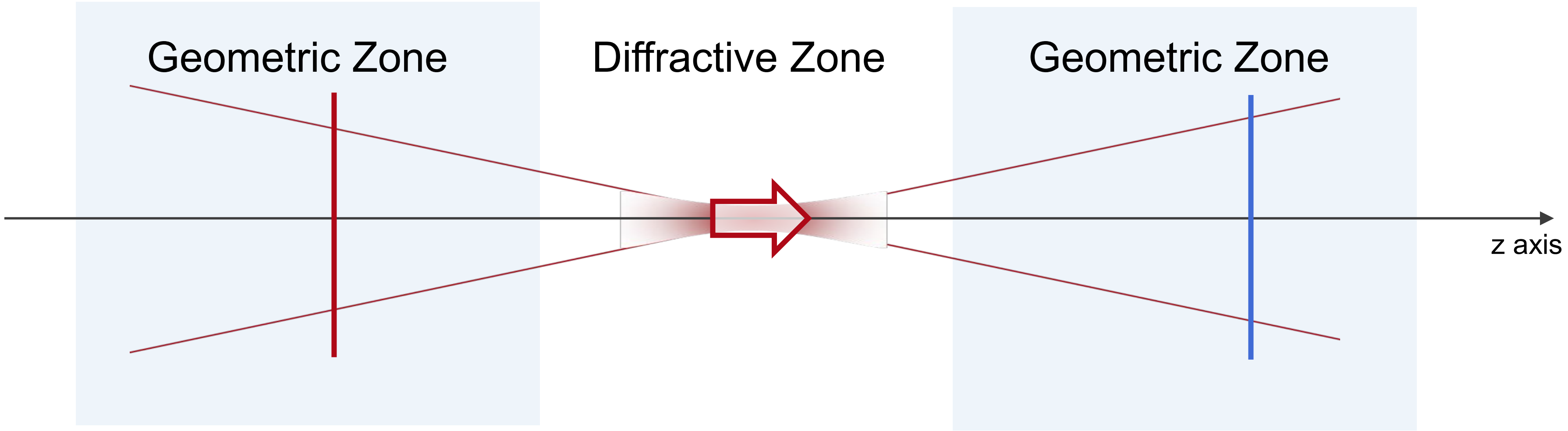

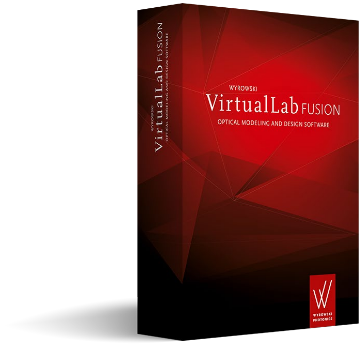

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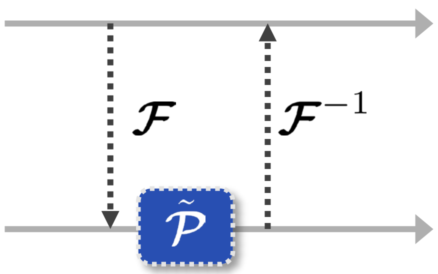

2019


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Journal of the Optical Society of America A

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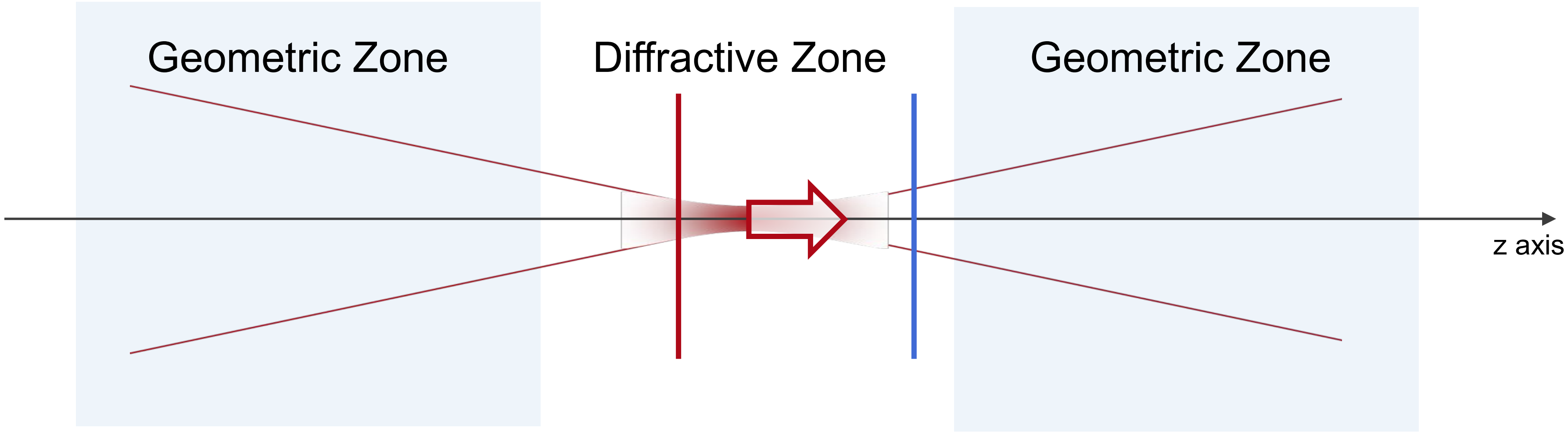
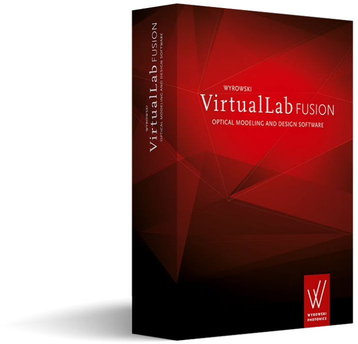
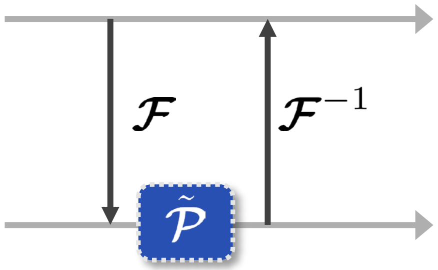
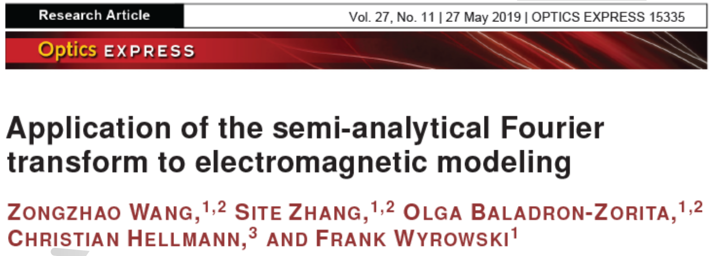




Fast equivalent to Rayleigh-Sommerfeld integral



2019

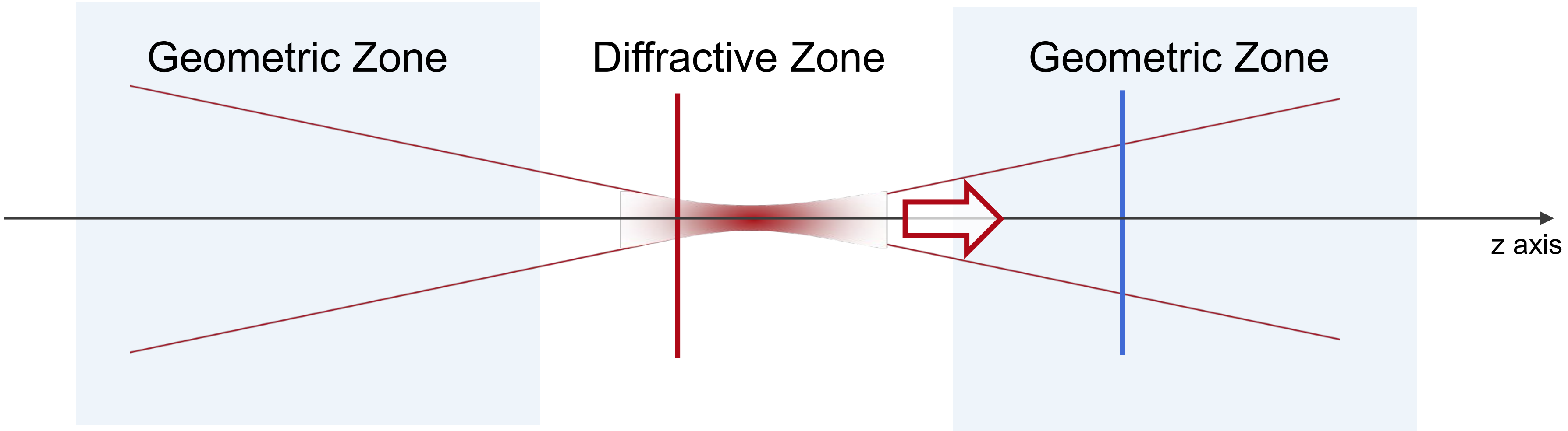

Research Article Vol. 27, No. 11 | 27 May 2019 | OPTICS EXPRESS 15335

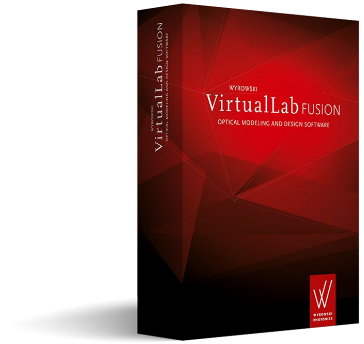
Optics EXPRESS

Application of the semi-analytical Fourier transform to electromagnetic modeling

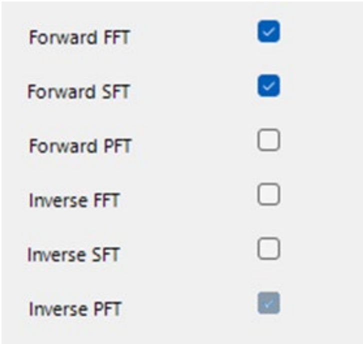
ZONGZHAO WANG,[1,2] SITE ZHANG,[1,2] OLGA BALADRON-ZORITA,[1,2] CHRISTIAN HELLMANN,[3] AND FRANK WYROWSKI[1]

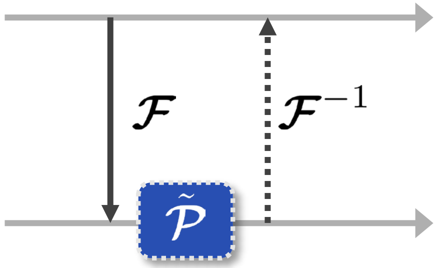
Propagation Scenarios and Selection of Fourier Transform





Generalized
Far-Field integral





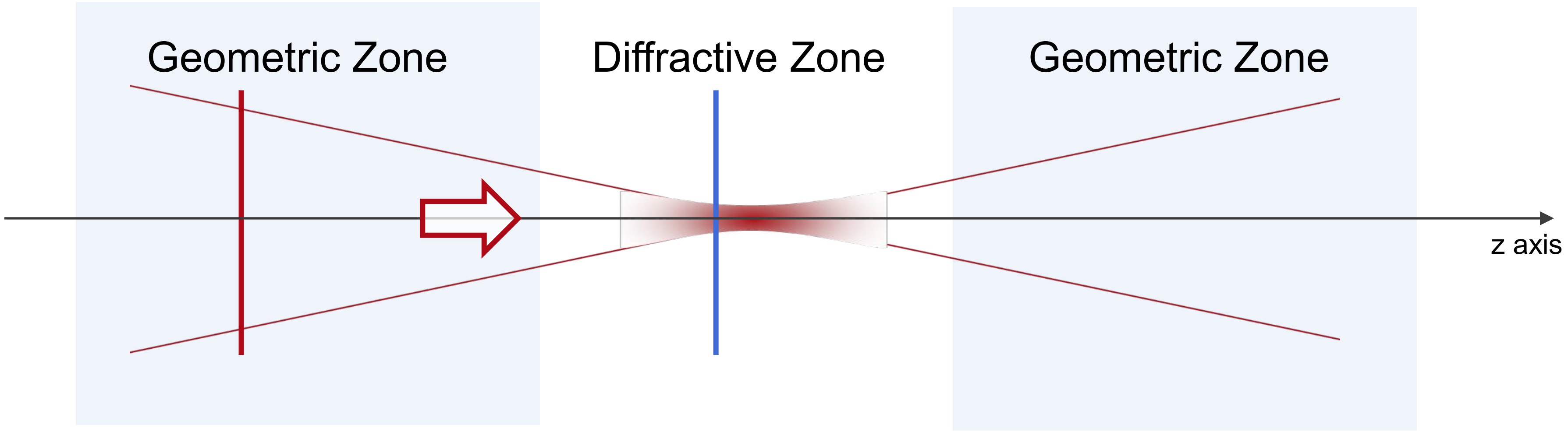
2021

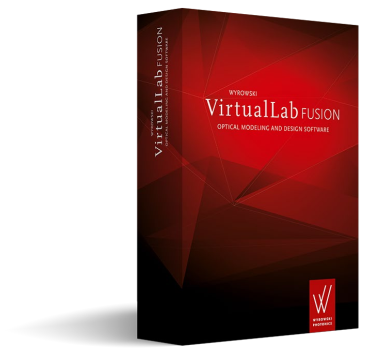
Research Article Vol. 29, No. 2 / 18 January 2021 / Optics Express 1774
Optics EXPRESS

Generalized far-field integral

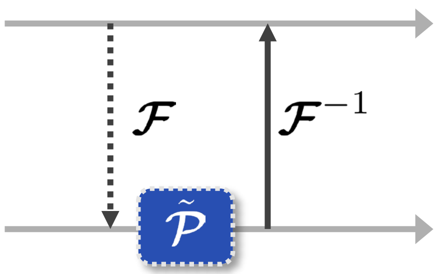
ZONGZHAO WANG,[1,2,*] OLGA BALADRON-ZORITA,[1,2] CHRISTIAN HELLMANN,[3] AND FRANK WYROWSKI[1]

Propagation Scenarios and Selection of Fourier Transform





Generalized Debye integral



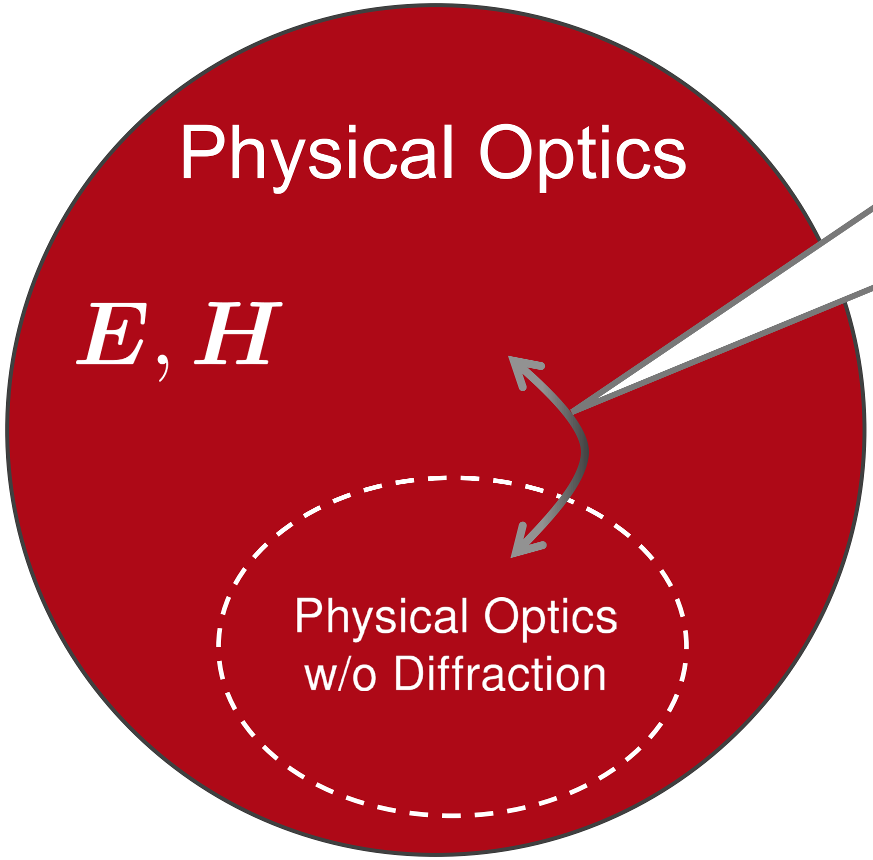
2020


Research Article Vol. 28, No. 17 / 17 August 2020 / Optics Express 24459
Optics EXPRESS

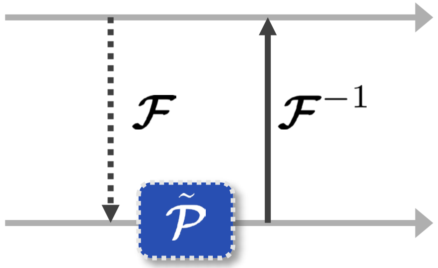
Generalized Debye integral

ZONGZHAO WANG,[1,2,*] OLGA BALADRON-ZORITA,[1,2] CHRISTIAN HELLMANN,[3] AND FRANK WYROWSKI[1]

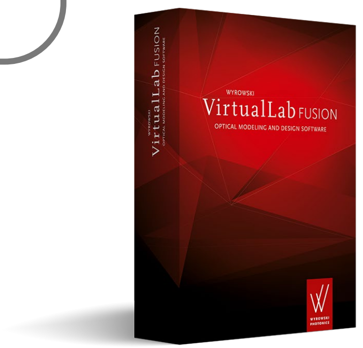
Geometrical Optics for Electromagnetic Fields

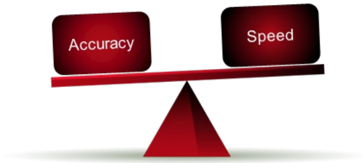


Enables the seamless inclusion of diffraction effects within a physical optics simulation framework.



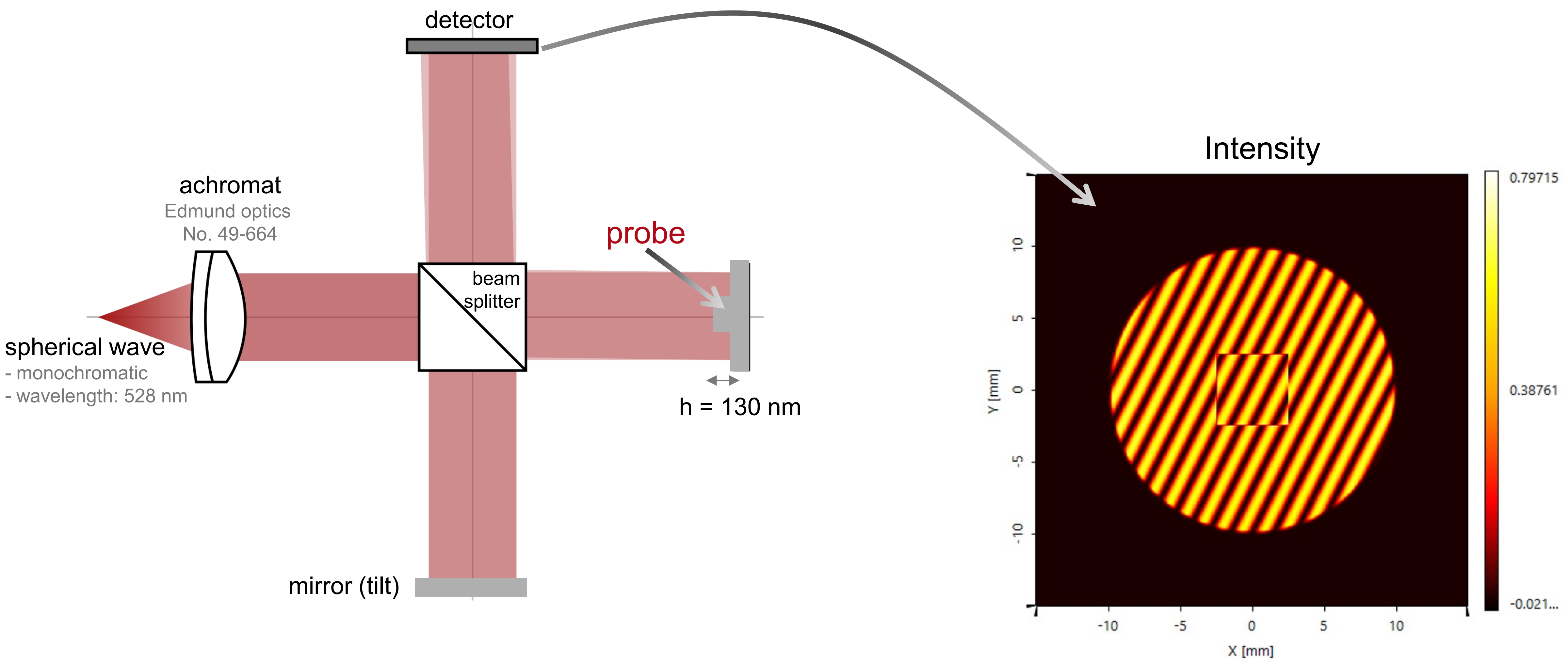
Seamless control by manual or automatic selection of Fourier transform algorithms!



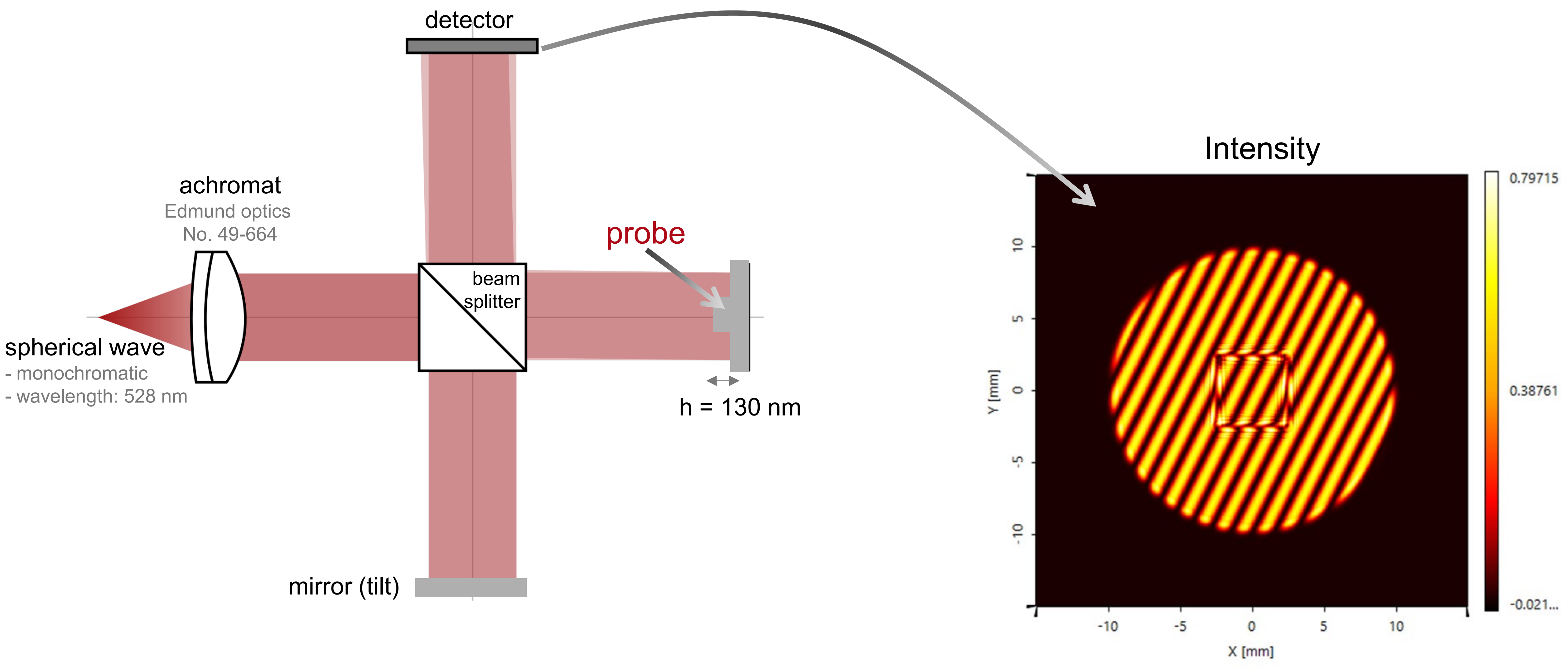


Control of accuracy-speed balance

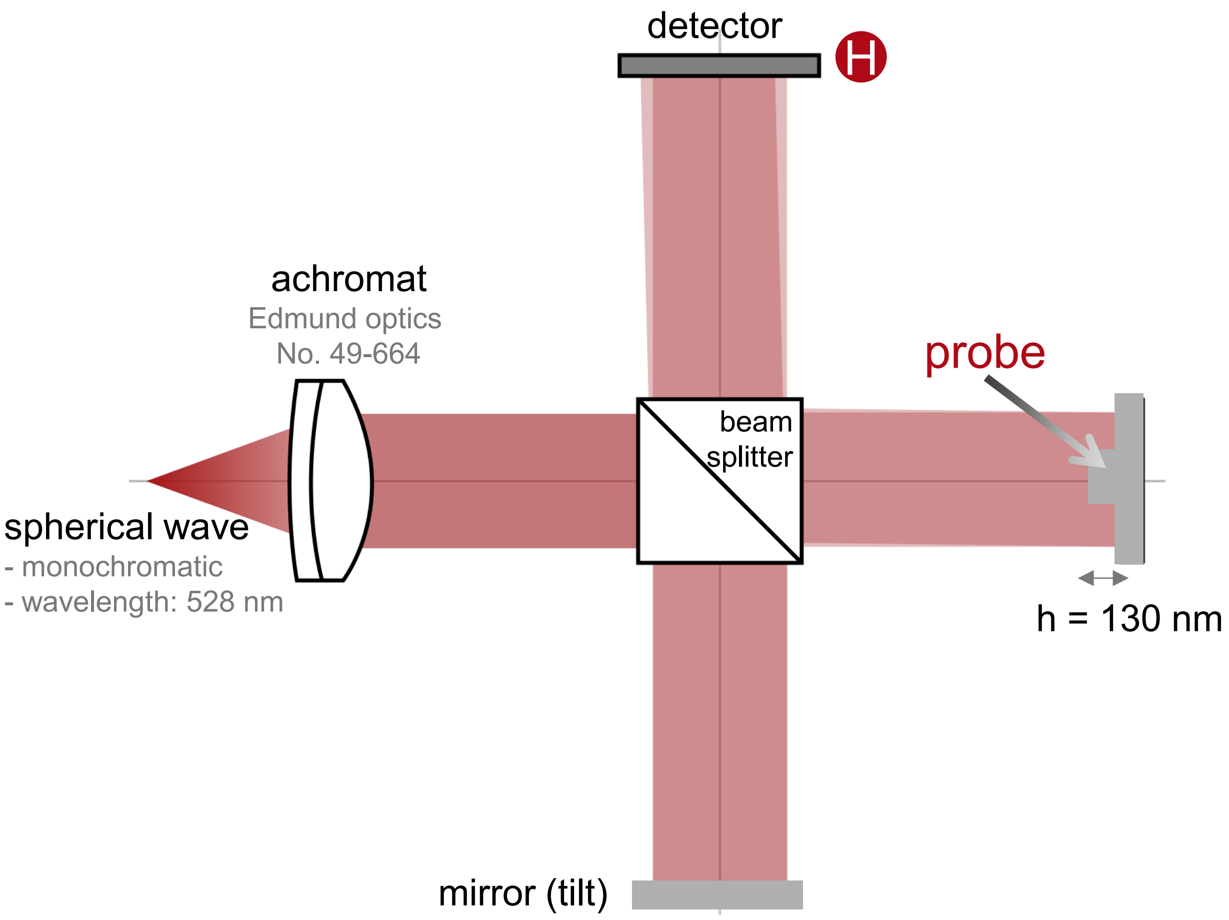
Interferometer Modeling: Pointwise FT Algorithm Only

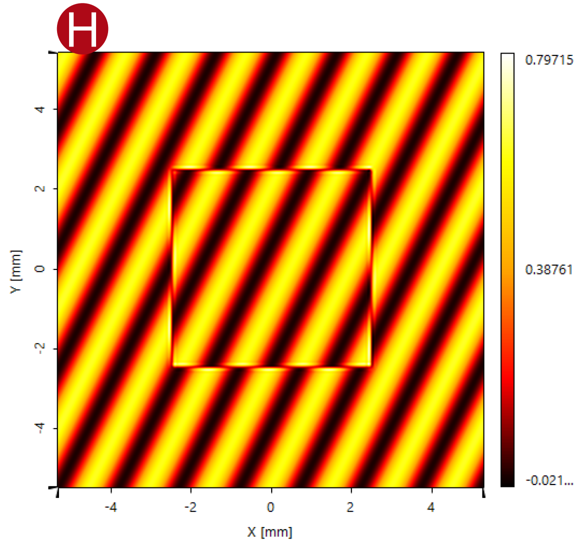


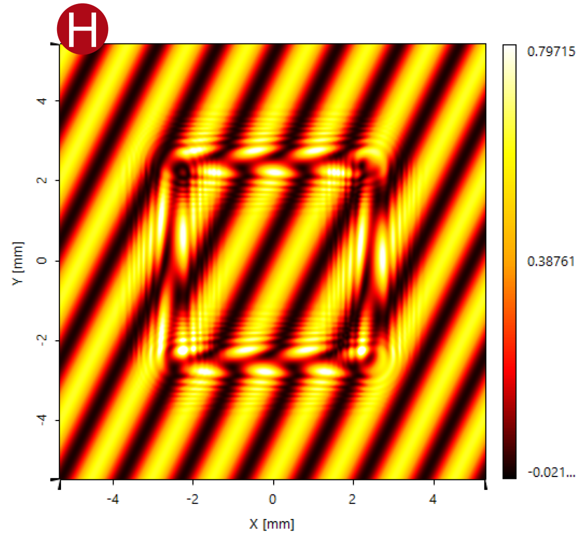
Interferometer Modeling: Automatic Selection of FT Algorithms



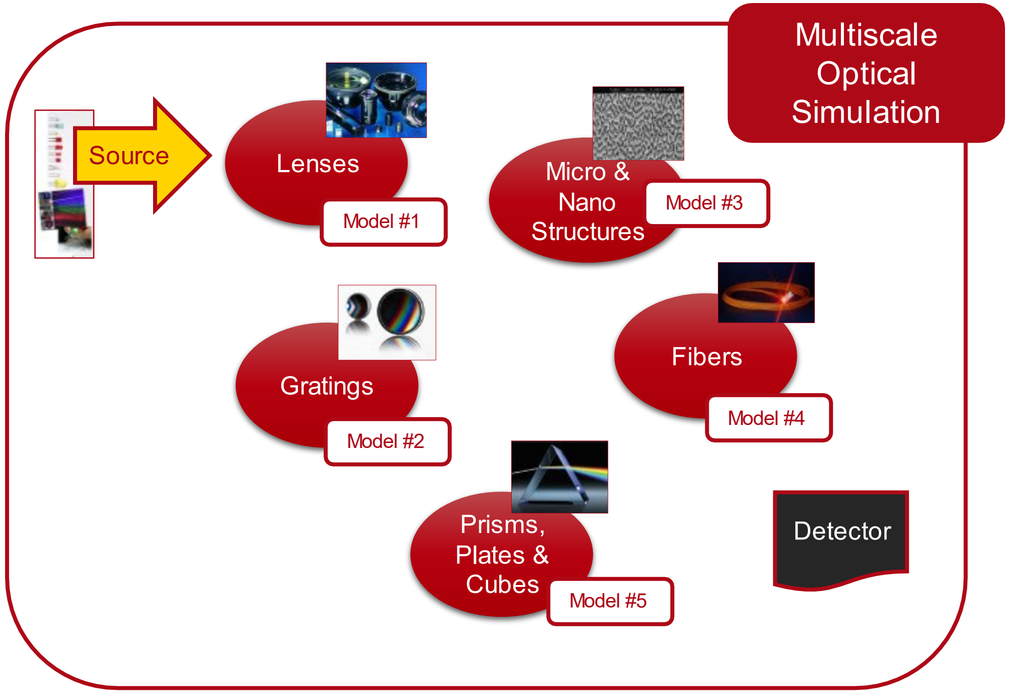
Seamless Inclusion of Diffraction Effects



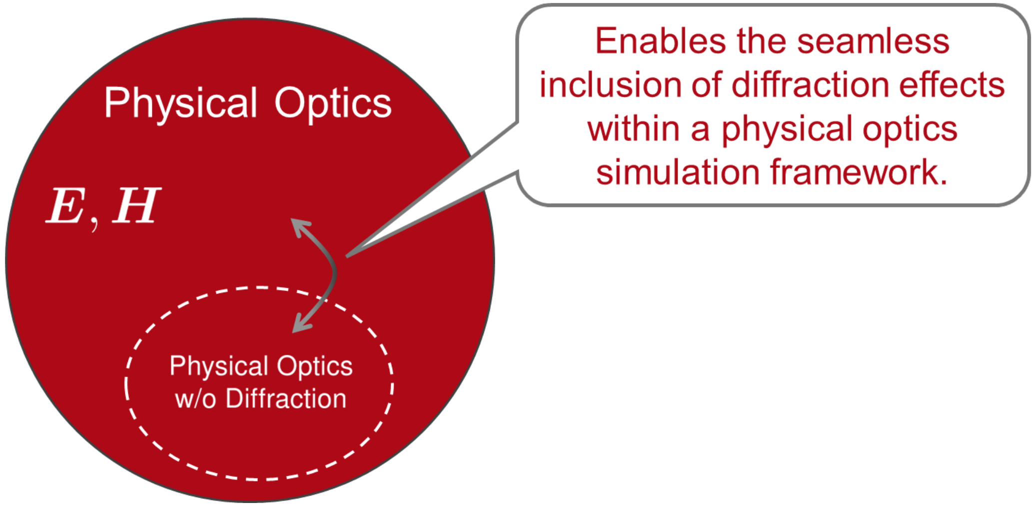


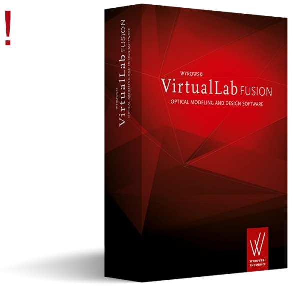


Megatrend Multiscale Optical Simulation and Design



Requires generalization of geometrical optics in **theory and practice**!





We need to identify that part of physical optics, which deals with the "***geometrical laws relating to the propagation of the 'amplitude vectors' E and H.***"

Citation
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